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Full Length Article

# Hyperinflation potential in commodity-currency trading systems: Implications for sustainable development

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ABSTRACT

Sustainable Development implies slowing the rate of utilization and eventual depletion of non-renewable resources such as oil and metals. Non-renewable resources are now commonly traded, often as derivatives, through electronic trading exchanges and studies the impact of that trading on sustainable development are underrepresented. Commodity-currency research since 2003 to some extent has focused on the relationship between commodity prices – including non-renewable resources – and the exchange rates of the currencies of the nations that are extracting those commodities. To a lesser extent, other research on non-renewable resource development has focused on technology and innovation. Here we address one issue at the core of non-renewable sustainable development: the question of commodity-currency linkages and spillovers and their effects on price stability. Our research tool is an economic interpretation of the Lotka-Volterra equations. Using Lotka-Volterra parameters from the fit to actual CAD XCT data, we find that carrying out the currency-commodity dynamics over several centuries demonstrates the possibility for cyclical unsustainable hyperinflation. Devaluing or inflating the currencies or commodities is not a solution to hyperinflation. The solution to hyperinflation is to increase $\beta$ (the effect of the commodity on the currency) and/or $\delta$ (the price decrease rate of the commodity when the currency is absent) and/or decrease $\gamma$ (the inflation rate of the commodity in the presence of the currency).



## Introduction

Sustainable Development implies slowing the rate of utilization and eventual depletion of non-renewable resources such as oil and metals (Moldan, Janoušková, & Hák, 2012). This need to manage non-renewable resource consumption is captured in three of the United Nation's Sustainable Development Goals (SDGs; UN DESA 2021): Goal 7 (Affordable and Clean Energy), Goal 8 (Decent Work and Economic Growth), and Goal 13 (Climate Action). Many studies on meeting these three goals are high-level and cross-sectional analyses for policy making, e.g., human capital decreases the consumption of non-renewable energy (Alvarado et al., 2021), economic growth and non-renewable energy utilization significantly accelerate the environmental deficit, (Usman, Khalid, & Mehdi, 2021), and non-renewable energy and urbanization contribute to the rise in emissions. (Wang et al., 2021). Other studies have focused on specific material substitutes (e.g., Carretero-Gómez & Piedra-Muñoz, 2021).

Non-renewable resources are now commonly traded, often as derivatives, through electronic trading exchanges and studies the impact of that trading on sustainable development are underrepresented. Trading has shaped research, because trading entails attempting to forecast future prices. To that end, commodity-currency research since 2003 (Chen & Rogoff, 2003; Chevalier & Ielpo, 2013b) has focused to some extent on the relationship between commodity prices – including non-renewable resources – and the exchange rates of the currencies of the nations that are extracting those commodities. This work has been focused on whether commodity values lead currency values or vice versa (e.g., Chen & Rogoff, 2003, Rogoff, Rossi, & Chen, 2008, Clements & Fry, 2008, Chen, Rogoff, & Rossi, 2010, Chen & Lee, 2018). Though this research has been for purposes of forecasting and not for sustainable development, it provides key insights for sustainability: once we understand how these systems work, we can better control them for purposes of sustainable development.

To a lesser extent, other research on non-renewable resource development has focused on technology and innovation. Research on innovation focuses on modelling the impact of using revenues from non-renewable resource extraction to engage in technological innovation such as material substitutes (Carretero-Gómez & Piedra-Muñoz, 2021); or using those revenues to innovate in non-resource areas to diversify the economy and increase opportunities

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(Lafforgue, 2008, Le & Le Van, 2018). Research on the sustainable development of non-renewable resources focuses on controlling extraction rates (Allen & Day, 2014, Dobra, Dobra, Ouedraogo, 2018) such as through taxes (Day & Day, 2019).

Here we address one issue at the core of non-renewable sustainable development: the question of commodity-currency linkages and spillovers and their effects on price stability. Relatively stable prices and a stable economy are likely a prerequisite for sustainable development. For our investigation we utilize a model rather than statistical techniques like correlation (e.g., Yang, Cai, & Hamori, 2018). Using daily nominal currency exchange rates and non-renewable commodity market value data from 1996 to 2020, we show that exchange trading has introduced the potential for economically disruptive ultra-long hyperinflation waves. Our research tool is an economic interpretation (Palomba, 1939, Gandolfo, 2008) of the Lotka-Volterra equations (Lotka, 1920, 2002, Volterra, 1926).

Our study is relevant to the sustainable use of non-renewable resources, as mediated by commodity and currency exchange traders. It is relevant to those seeking to understand and mitigate risk in commodity-currency systems.

We proceed with a discussion of the theory of commodity and currency cycles, followed by a discussion of the Lotka-Volterra equations. We continue with a discussion of the method used in this research. We then present the results, a discussion of the results, and a conclusion.

## Theoretical background

### *Commodity and currency cycles*

Previous studies analyze commodity cycles (Kablan, Ftiti, & Guesmi, 2017; Drechsel & Tenreyro, 2018, Jacks, 2013, Agnello, Castro, Hammoudeh & Sousa, 2020, Jacks & Stuermer, 2020) and super-cycles (Cuddington & Zellou, 2013, Erten & Ocampo, 2013, Erdem & Ünalmış, 2016) and currency cycles (Lin, Lin, & Wu, 2015). Studies on such cycles in commodity-currency systems qua systems are underrepresented, i.e., though currency hyperinflation is a perennial topic, studies on commodities hyperinflation are underrepresented.

### *Lotka-Volterra*

Why Lotka-Volterra? In previous work (Marinakis, White, & Walsh, 2020), currency pairs appeared to behave like coupled oscillators. The Lotka-Volterra equations were applied to daily ASEAN+5-3 currency pair exchange rates using the Swiss franc as the numaire. The equations fit broad-scale (multi-year) patterns, which suggested the potential for much longer-term – and higher amplitude – dynamics in exchange rates arising from endogenous nonlinear interactions.

An economic interpretation of Lotka-Volterra enables application of the equations to currency-commodity pairs (Palomba, 1939, as discussed by Gandolfo, 2008). We now discuss the Lotka-Volterra equations and their economic interpretation.

The Lotka–Volterra Eqs. (1), (2) are

$$\frac{dx(t)}{dt} = x(t)\,(\alpha - \beta y(t)) \tag{1}$$

$$\frac{dy(t)}{dt} = -y(t)(\delta - \gamma\, x(t)) \tag{2}$$

with $\alpha > 0$, $\beta > 0$, $\gamma > 0$, $\delta > 0$, $x(0) > 0$, $y(0) > 0$.

In the original Lotka-Volterra system (Lotka, 1920, 2002, Volterra, 1926), the variable x(t) is the population size of the prey at time t and the variable y(t) is the population size of the predator at time t. The parameter $\alpha$ is the birth rate of the prey when the predator is absent (i.e., the natural growth rate of the prey), $\delta$ is the death rate of the predator when the prey is absent, $\beta$ is the effect of predation on the prey, and $\gamma$ is the propagation rate of the predator in the presence of prey.

The Lotka-Volterra system oscillates at period $2\pi\,(\alpha^{*}\,\delta)^{-0.5}$. These oscillations are neutrally stable. The amplitude of those oscillations is also of interest, as a relatively large amplitude in our case would constitute hyperinflation. It is commonly stated that amplitude in the Lotka-Volterra equations depends on the initial conditions, i.e., the initial values of x(t) and y(t). For three or fewer interacting species, these models cannot have chaotic solutions or limit cycles (Roques & Chekroun, 2011).

Palomba (1939), as discussed by Gandolfo (2008), provides an economic interpretation of Lotka-Volterra. He interprets prey as goods ready for immediate consumption (and goods that directly enter into their production). He interprets predators as capital goods that directly enter into the production of other capital goods (and only indirectly into the production of final goods). The economy tends to increase capital goods and diverts goods ready for immediate consumption towards capital goods, i.e., it is necessary to reduce consumption in order to invest in capital goods. Absent this diversion, goods ready for immediate consumption would increase and capital goods would decrease to zero. Lotka-Volterra captures this dynamic by its four parameters (Table 1). Goods ready for immediate consumption have a coefficient of increase of $\varepsilon_1$ and capital goods that directly enter into the production of other capital goods have a coefficient of increase of -$\varepsilon_2$, where $\varepsilon_1$ and $\varepsilon_2$ are positive constants. Goods ready for immediate consumption have a coefficient of decrease of -$\gamma_1$ and capital goods that directly enter into the production of other capital goods have a coefficient of increase of $\gamma_2$, where $\gamma_1$ and $\gamma_2$ are positive constants. If x(y) is the volume of goods ready for immediate consumption at time t and y(t) is the volume of capital goods at time t:

- $\alpha$ is the coefficient of increase of goods ready for immediate consumption,
- -$\beta$ is the coefficient of decrease of goods ready for immediate consumption,
- -$\gamma$ is the coefficient of increase of capital goods, and
- -$\delta$ is the coefficient of increase of capital goods, where

**Table 1**
Comparison of interpretation of Lotka-Volterra as applied to commodities-currencies against Original Lotka-Volterra parameters.

| | Parameter $\alpha$ | Parameter -$\beta$ | Parameter -$\gamma$ | Parameter $\delta$ |
|---|---|---|---|---|
| Original Lotka-Volterra | birth rate of the prey when the predator is absent | effect of predation on the prey | propagation rate of the predator in the presence of prey | death rate of the predator when the prey is absent |
| Lotka-Volterra as applied to commodities-currencies | coefficient of increase of currency exchange rate (endogenous component of exchange rate) | coefficient of decrease of the currency exchange rate (exogenous component of exchange rate) | coefficient of increase of the commodity (influence of currency on commodity; exogenous component of commodity) | coefficient of increase of capital goods (natural price decrease rate of commodity; endogenous component of commodity) |

$\alpha$x indicates exponential growth of goods ready for immediate consumption,
$\beta$xy is the loss rate of goods ready for immediate consumption (biologically, the rate at which predators y and prey x meet, or the rate of predation)
$\gamma$y is the loss rate of capital goods (biologically, through death or emigration, or intrinsic mortality rate)
$\delta$xy is the growth rate of capital goods (biologically, the rate at which predators y consume prey x)

In terms of commodity-currency dynamics, given Palomba's interpretation, currencies are more appropriately considered the goods ready for immediate consumption, or x(t), or the prey; and commodities are the capital goods, or y(t), or the predator. In Lotka-Volterra, prey precede predators, such that currencies precede commodities. The latter is a minority view (Amano & Van Norden, 1995) and is considered outdated in light of more recent work, e.g., Clements and Fry (2008). We discuss this below in our Results section. Palomba's interpretation suggests that, instead of only buying money (a good ready for immediate consumption) and pushing up its value, people to some extent are diverting their money to buy commodities (which is a capital good):

$\alpha$ is the coefficient of increase of currency exchange rate,
-$\beta$ is the coefficient of decrease of the currency exchange rate,
-$\gamma$ is the coefficient of increase of the commodity, and
$\delta$ is the coefficient of increase of the commodity.

The Lotka-Volterra equations, as a model of the currency-commodity system, are epistemologically related to the explanation of the system (Rohwer and Rice, 2016). Rather than metaphysically providing an explanation of the system, it produces a scientific understanding (of the system) that is important to the discovery of an explanation (of the system) – without actually providing an explanation or perhaps even a partial explanation (of the system). The model provides an understanding of the system without actually providing the explanation. The four Lotka-Volterra parameters do not necessarily represent anything real. Outside of the realms of statistics and models, there might not be such things as an endogenous or an exogenous component of an exchange rate. However, the parameters provide us with a way to think about the system dynamics. We now turn to a discussion of the four Lotka-Volterra parameters.

*Parameter $\alpha$: inflation rate of the currency when the commodity is absent (endogenous component of exchange rate; natural inflation rate of the currency)*

In a study using quarterly data from 1975 to 2005 of International Monetary Fund commodity price indices and exchange rates, it was shown that spillovers from currencies to commodities (1%) were less than than spillovers from commodities to currencies (2 % to 5.2 %), but the contributions were small (Clements & Fry, 2008); this suggested that both commodity prices and currency exchange rate were mainly endogenous. Researchers performing factor analyses on exchange rate returns have found a two-factor structure comprising a U.S. dollar factor and a euro factor, where commodity exporting countries load positively on the dollar factor (Greenaway-McGrevy, Mark, Sul, & Wu, 2018). This suggests that the dollar and the euro play important roles in exchange rate determinations. Other research showed that exchange rates in emerging markets were determined primarily by local components such as economic conditions, political situations, and institutional designs (Liu, Wang, Wang, & Zhang, 2019). Interest rate factors help explain exchange rate fluctuations (Yung, 2021). National monetary policies are often focused on controlling inflation rates (García & Mejía, 2018).

*Parameter $\beta$: effect of the commodity on the currency (exogenous component of exchange rate)*

The Lotka-Volterra nonlinear terms $\beta$xy and $\delta$xy explicitly account for interactive or spillover effects. However, the parameter $\beta$ alone implicitly accounts for spillovers from the commodity to the currency. Many studies have examined the spillover effects between the equities markets and commodities. For example, oil futures are neutral but gold futures are net recipients of spillover shocks from equities markets (Yoon, Al Mamun, Uddin, & Kang, 2019). Equities markets are net volatility transmitters to oil markets (Gomez-Gonzalez, Hirs-Garzón, & Sanín-Restrepo, 2021).

Prior to 2003, commodity currency exchange rates were treated as buffers between exogenous effects on commodity values and on the commodity-producer economies. In 2003 it was then shown that some commodity prices had "a strong and stable influence" on some real exchange rates (Chen & Rogoff, 2003: 155). Seven years later, the same research team showed the opposite, that "exchange rates of a number of small commodity exporters have remarkably robust forecasting power over global commodity prices" both in- and out-of-sample and that "commodity prices Granger-cause exchange rates in-sample" (Chen et al., 2010: 1145). This later study used quarterly data and their analyses were on the scale of quarters. Other researchers (Groen & Pesenti, 2011) subsequently performed a regression study that used commodity currencies to forecast global commodity indices. This study also used quarterly data. The regression model, for one-quarter forecasts, outperformed a random walk but not an autoregessive model; and vice versa for one-year forecasts.

In other studies, oil was shown to be a net transmitter of shocks to currencies (Albulescu, Demirer, Raheem, & Tiwari, 2019). Volatility spillovers of industrial metals were shown to affect the evolution of the Australian dollar, the New Zealand dollar, and the Mexican peso (Chevallier & Ielpo, 2013a). Global market power can reduce the exchange rate response to global commodity price shocks (Chen & Lee, 2018).

*Parameter $\gamma$: inflation rate of the commodity in the presence of the currency (influence of currency on commodity)*

One study found strong causal effects mainly from currencies to commodities, with bidirectional causality between gold and the New Zealand dollar, Brent oil and the Brazilian real, and copper and the Chilean peso (Belasen & Demirer, 2019). The study used a GARCH (1,1) model on daily futures prices from 2007 to 2016. This finding contradicts the findings of Chen & Rogoff (2003) and subsequent studies.

*Parameter $\delta$: price decrease rate of the commodity when the currency is absent (endogenous component of the commodity price; natural price decrease rate of commodity)*

In a study of 12 agricultural goods, metals, and soft commodities from 1870 to 2013, after accounting for long-run trends in real commodity prices, it was found that demand shocks explain on average 35% of the price change whereas supply shocks explain on average 20% of the price change (Jacks & Stuermer, 2020).

## Methods

We proceed as follows. We first obtain a number of fits of the Lotka-Volterra model to currency-commodity pairs to demonstrate feasibility and applicability. We then demonstrate the potential for high amplitude (E3-to E9-order of magnitude spikes) ultra-long (100- to 500-year long periods) oscillations, i.e., unsustainable hyperinflation, in several of those Lotka-Volterra model fits. We then systematically vary Lotka-Volterra parameter values for one currency-commodity pair, the Canadian dollar (CAD) and West Texas Intermediate crude barrel (XCT), to identify the types of interventions that would be necessary to decrease the amplitude and period of the Lotka-Volterra oscillations, i.e., of the ultra-long hyperinflation waves.

*Data*

We constructed time series for daily nominal currency exchange rates for select commodities and for the so-called commodity currency prices. The selected commodities were silver (symbol XAG), gold (symbol XAU), platinum (symbol XPT), Brent Crude barrel (XCB), and West Texas Intermediate as crude (barrel) (XCT). The commodity currencies were from Australia, Brazil, Chile, New Zealand, Norway, Russia, and South Africa. We used the Swiss franc as the base currency or numaire because it is a leading safe haven and it is not a commodity currency. The date range extended from August 6, 1996 to February 18, 2020. We deleted days from all data sets if they were not common to all eight data sets, resulting in N = 5445. Data was obtained from the PACIFIC Exchange Rate Service at the University of British Columbia Sauder School of Business (Antweiler, 2020).

As a preliminary matter we performed ARIMA($p$,$q$,$d$) characterizations of the time series. Integrated order $d$ was determined by Dickey-Fuller Unit Root tests and Seasonal Augmented Dickey-Fuller Unit Root tests. Autoregressive order $p$ and Moving Average order $q$ were determined by the traditional means, i.e., by examining autocorrelations and by performing Autocorrelation Check of Residuals of candidate models.

*Lotka-Volterra*

We attempted to fit all currency and commodity pairs with the Lotka-Volterra model, in all combinations, with both commodities leading (i.e., as prey) currencies (as predators) and currencies leading commodities. We parsimoniously used the original and simplest form of Lotka-Volterra. For completeness we also attempted to fit a three-dimensional Lotka-Volterra model (Hsu, Ruan, & Yang, 2015).

We systematically varied the initial values as well as the parameters in one case, CAD XCT, to investigate whether and how these affect the dynamics.

Both static and dynamic fits of the Lotka-Volterra equations were obtained. All computations were performed using SAS (2016). We utilized static fits to see how closely we could fit the equations to the data. We utilized dynamic fits to see how closely the equations could be used to forecast the data, since dynamic fits are based only on initial data points.

**Table 2**
Commodity-currency pairs that had statistically significant parameters and non-trivial R-squares are marked with an X.

| Currency / Commodity | XCB | XCT | XPT | XAU | XAG |
|---|---|---|---|---|---|
| AUD | | | | | X |
| BRL | X | X | X | X | X |
| CAD | | X | | | |
| CLP | | | | | |
| NZD | X | X | | | X |
| NOK | | | | | |
| RUB | | | | | X |
| ZAR | | | | | |

## Results

Each time series was best fit by an ARIMA(7,1,0) model (not shown). We were able to fit the two-dimensional Lotka-Volterra equations only to a subset of the commodity-currency pairs and only with currencies leading commodities (Table 2; Figs. 1-15). Commodities play significant roles in the economies of the subset of commodity-currency pairs. No three-dimensional fits were achieved, *e.g.*, CAD NZD XCT. We did not find the forementioned (Belasen & Demirer, 2019) bidirectional causality between gold and the New Zealand

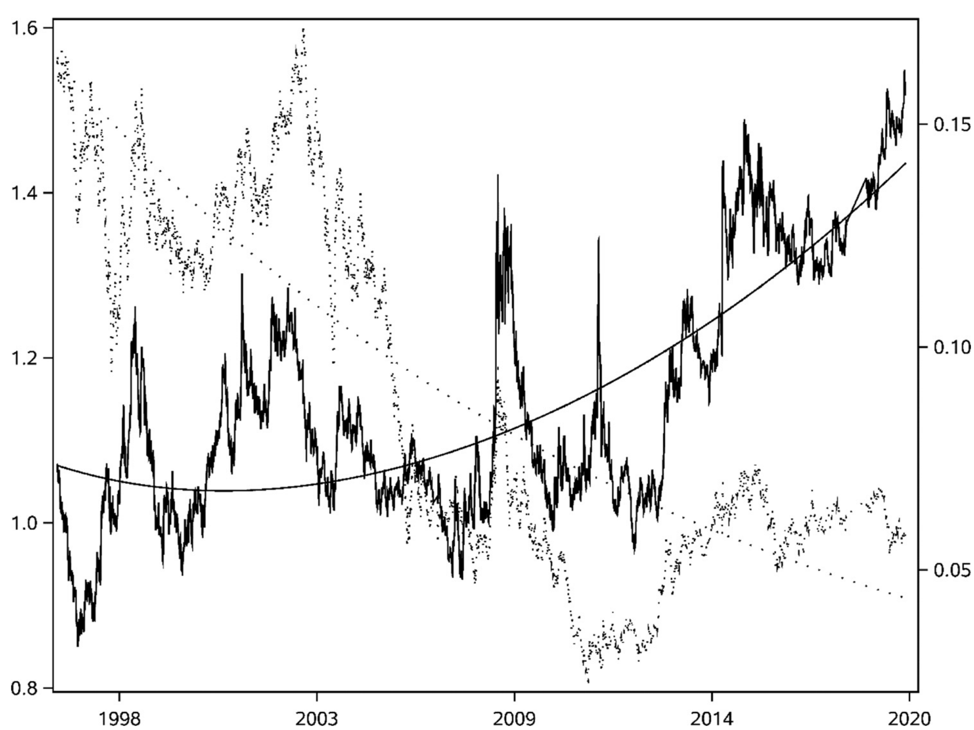


**Fig. 1.** AUD XAG time series plot. Solid lines are the currency and dotted lines are the commodity. Jittery lines show the static solution and smooth lines show the dynamic solution. Parameter values and R-squares are shown in Tables 3 and 4. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

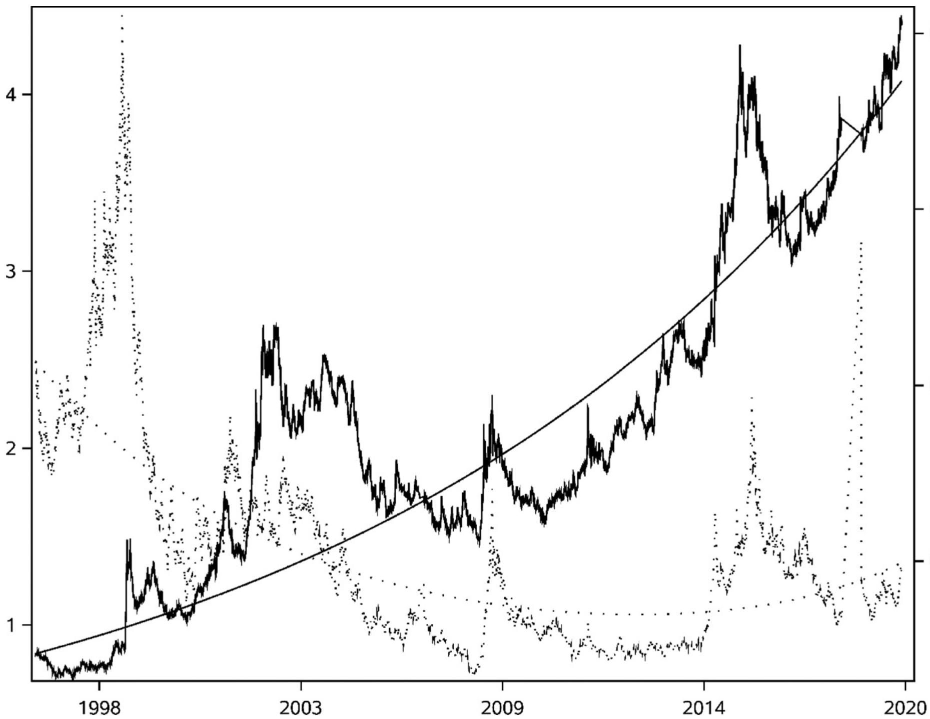


**Fig. 2.** BRL XCB time series plot. Solid lines are the currency and dotted lines are the commodity. Jittery lines show the static solution and smooth lines show the dynamic solution. Parameter values and R-squares are shown in Tables 3 and 4. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

dollar, Brent oil and the Brazilian real, and copper and the Chilean peso. We now turn to the parameter values (Tables 3, 4).

*Parameter Values*

Though the literature suggests commodities lead currencies, such that parameter $\beta$ (the effect of commodities on currencies) should be greater than parameter $\gamma$ (the effect of currencies on commodities), we found the opposite as reflected in the medians of the dynamic fit parameters (Table 4). This finding corroborates our inability to fit any commodity-currency pairs in which the commodity led the currency. Previous studies analyzed Granger causality on daily (Belasen & Demirer, 2019) or quarterly time scales (Chen & Rogoff, 2003, Rogoff et al., 2008, Clements & Fry, 2008, Chevallier & Ielpo, 2013a, Chen & Lee, 2018). Our finding that currencies lead commodities may be related to the longer temporal scale of our fit of the Lotka-Volterra equations, i.e., years.

The instances in which the parameter values were zero – or 1E-8 – were common in the dynamic fits. Only one pair, CAD XCT, did not have at least one parameter value $\alpha$, $\beta$, $\gamma$, $\delta$ that was equal to 1E-8. The 1E-8 parameter values were obtained because the logic of the differential equation algorithm tried to set those parameter values to zero but we restricted against it per the assumptions of the Lotka-Volterra model. Different parameters were equal to 1E-8 for different currency-commodity pairs, indicating not that the model was inadequate but simply that the term did not play a role in that commodity-currency interaction. Specifically, in the dynamic fits:

- Parameter $\alpha$, the inflation rate of the currency when the commodity is absent (the endogenous component of exchange rate), had no parameter values equal to zero (1E-8).
- Parameter $\beta$, the effect of the commodity on the currency (the exogenous component of exchange rate), was zero (1E-8) for all BRL pairs and for RUB XAG. This suggests that the independence

**Table 3**
Lotka-Volterra parameters from static fits. Parameter values of 1E-8 were not statistically significant (the algorithm wanted to set them to zero but we restricted against it). BRL XAU $\beta$, $\gamma$, $\delta$ were not statistically significant. All other parameters were statistically significant at p=0.05 and often p<.0001.

| Currency Commodity | Parameter $\alpha$ | Parameter $\beta$ | Parameter $\gamma$ | Parameter $\delta$ | X R-square | Y R-square | Period (years) |
|---|---|---|---|---|---|---|---|
| AUD XAG | 0.000342 | 0.00317 | 0.000575 | 0.000211 | 0.99 | 0.99 | 64.1 |
| BRL XCB | 0.000547 | 0.023989 | 0.002228 | 0.002477 | 0.99 | 0.99 | 14.8 |
| BRL XCT | 0.000528 | 0.022554 | 0.002439 | 0.002512 | 0.99 | 0.99 | 14.9 |
| BRL XPT | 0.000251 | 0.092487 | 1E-8 | 0.002981 | 0.99 | 0.87 | 19.9 |
| BRL XAU | 0.00043 | 0.239875 | 1E-8 | 0.002563 | 0.99 | 0.98 | 16.4 |
| BRL XAG | 0.000362 | 0.005648 | 0.002325 | 0.001404 | 0.99 | 0.99 | 24.1 |
| CAD XCT | 0.000151 | 0.006476 | 0.002841 | 0.002377 | 0.99 | 0.99 | 28.7 |
| NZD XCB | 0.000076 | 0.004837 | 0.000312 | 0.000017 | 0.99 | 0.99 | 478.9 |
| NZD XCT | 0.000076 | 0.004684 | 0.000556 | 0.000285 | 0.99 | 0.99 | 117.0 |
| NZD XAG | 0.000138 | 0.001937 | 0.002027 | 0.001374 | 0.99 | 0.99 | 39.5 |
| RUB XAG | 0.00037 | 1E-8 | 0.0009 | 0.000037 | 0.99 | 0.99 | 147.1 |
| Mean | 0.000297 | 0.036878 | 0.001291 | 0.001476 | | | |
| Median | 0.000342 | 0.005648 | 0.0009 | 0.001404 | | | |

**Table 4**
Lotka-Volterra parameters from dynamic fits. Parameter values of 1E-8 were not statistically significant (the algorithm wanted to set them to zero but we restricted against it). All other parameters were statistically significant at p=0.05 and often p<.0001.

| Currency Commodity | Parameter $\alpha$ | Parameter $\beta$ | Parameter $\gamma$ | Parameter $\delta$ | X R-square | Y R-square | Period (years) |
|---|---|---|---|---|---|---|---|
| AUD XAG | 0.000122 | 0.000966 | 0.000154 | 1E-8 | 0.57 | 0.69 | 15585 |
| BRL XCB | 0.000184 | 1E-8 | 0.000474 | 0.000187 | 0.77 | 0.60 | 92.8 |
| BRL XCT | 0.000184 | 1E-8 | 0.000479 | 0.000201 | 0.77 | 0.59 | 89.5 |
| BRL XPT | 0.000184 | 1E-8 | 0.000536 | 0.000238 | 0.77 | 0.82 | 82.3 |
| BRL XAU | 0.000184 | 1E-8 | 0.000134 | 1E-8 | 0.77 | 0.74 | 12690 |
| BRL XAG | 0.000184 | 1E-8 | 0.000214 | 0.000032 | 0.77 | 0.71 | 224.3 |
| CAD XCT | 0.000209 | 0.008611 | 0.000143 | 0.001245 | 0.74 | 0.56 | 33.7 |
| NZD XCB | 0.000004 | 0.000676 | 0.000164 | 1E-8 | 0.35 | 0.42 | 86071 |
| NZD XCT | 0.000004 | 0.000677 | 0.000165 | 1E-8 | 0.35 | 0.36 | 86071 |
| NZD XAG | 0.000040 | 0.000175 | 0.000165 | 1E-8 | 0.35 | 0.71 | 27218 |
| RUB XAG | 0.000347 | 1E-8 | 0.000214 | 3.477E-6 | 0.77 | 0.71 | 495 |
| Mean | 0.00015 | 0.00101 | 0.000258 | 0.000173 | | | |
| Median | 0.000184 | 1E-08 | 0.000165 | 3.48E-06 | | | |

**Table 5**
Effects of Lotka-Volterra parameter changes on amplitude and period.

| | Parameter $\alpha$ | Parameter $\beta$ | Parameter $\gamma$ | Parameter $\delta$ |
|---|---|---|---|---|
| Decrease parameter | Amplitude decreased & period increased | Amplitude & period increased | Amplitude & period decreased | Amplitude & period increased |
| Increase parameter | Amplitude increased & period decreased | Amplitude & period decreased | Amplitude & period increased | Amplitude & period decreased |

of BRL from commodities can be an interesting subject of future study.

- Parameter $\gamma$, the effect of currencies on commodities (the inflation rate of the commodity in the presence of the currency), was never 1E-8. In light of the original Lotka-Volterra interpretation – the birth rate of the predator in the presence of the prey – it suggests that the commodity value is not entirely exogenous, *i.e.*, that the exchange rate may be contributing to the commodity value just as prey population numbers contribute to predator growth rates by functioning as food for the predator.
- Parameter $\delta$, the price decrease rate of the commodity when the currency is absent (the natural price decrease rate of commodity or the endogenous component of the commodity) was zero (1E-8) for all NZD pairs (with XCB, XCT, and XAG) and for AUD XAG and BRL XAU. This suggests that the commodity inflation rate in these cases is naturally positive definite.

### *Damping ultra-long hyperinflation wave*

We then investigated whether the Lotka-Volterra fits could demonstrate large amplitude cycles of commodity-currency pairs, i.e., cyclical commodity-currency hyperinflation. We investigated whether changing one or more of the parameters ($\alpha$, $\beta$, $\gamma$, $\delta$) could also change the amplitude. We chose to vary the parameters for one series, CAD XCT. We chose CAD XCT because it is the only pair that did not have a parameter equal to 1E-8 (Table 4) so we were able to manipulate all parameters. Using the parameters from the fit to actual data, we found that carrying out the currency-commodity dynamics over several centuries (Fig. 16) demonstrates the possibility for cyclical unsustainable hyperinflation. The phase portrait (Fig. 17) appears to show a limit cycle but this is actually the series pair relaxing into a periodic orbit from the initial values.

Contrary to what is commonly reported in the literature, we did not find that changing the initial value of x, i.e., x(0), could alone change the amplitude (Figs. 18-23). We were able to change the amplitude and the period by changing the parameters ($\alpha$, $\beta$, $\gamma$, $\delta$) (Table 5, Figs. 24-39).

**Table 6**
Dates of introductions of currencies and commodities trading.

| Currency or commodity | Date of Introduction |
|---|---|
| AUD | 14 February 1966 |
| BRL | 1 July 1994 |
| CAD | 1858 |
| CLP | 29 September 1975 |
| NZD | 1967 |
| NOK | 1875 (or 1914, following dissolution of the Scandinavian Monetary Union) |
| RUB | 1992, 1998 |
| ZAR | 1961 |
| XCB | 1976 |
| XCT | 28 January 1981 (date of US governmental decontrol of oil prices) |
| XPT | Arguably late 1980’s with the establishment of the London Platinum and Palladium Market (e.g., Bao 2020) |
| XAU | – |
| XAG | – |

## Discussion

### *Ultra-long wave hyperinflation*

By carrying out the Lotka-Volterra dynamics over several centuries using the parameters derived from fitting the equations to the actual data, we found the potential for longer-term patterns, namely ultra-long waves with 100- to 500-year long periods, resulting from nonlinear interactions between the commodities and the currencies. Moreover, these waves, were they to manifest, would bring cycles of 50- to 100-year hyperinflation on E3-to E9-order of magnitude followed by precipitous commensurate decline. We identified three cases to inform our response:

**Case 1.** We have not seen commodity-currency hyperinflation in the past or present as forecast by the model because the currencies are not yet old enough.

The forecast Lotka-Volterra cycles are of periods 80+ years (Table 4). The currencies and the commodities, as currently traded and in this study’s dataset, are younger than that, possibly with the exceptions of gold (XAU) and silver (XAG) (Table 6). Arguably all currencies date to the end of Bretton Woods on 15 August 1971. This case may be correct.

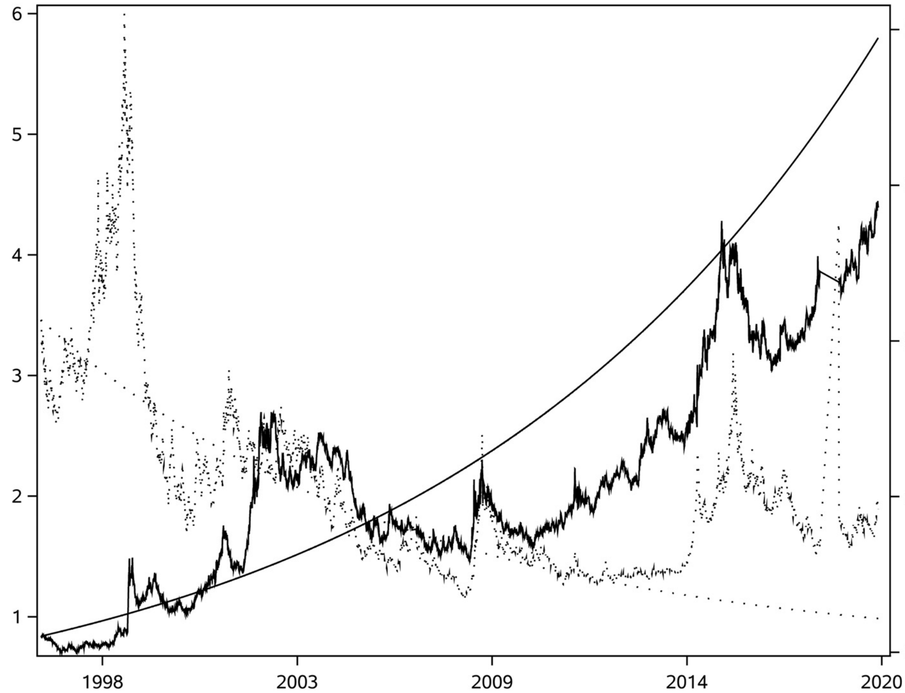


**Fig. 3.** BRL XCB time series plot showing parameter stability. Solid lines are the currency and dotted lines are the commodity. Compare to Fig. 2. Static Lotka-Volterra parameters were obtained using only the first half of the data and then used for the dynamic fit. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

**Case 2.** We have not seen commodity-currency hyperinflation in the past or present as depicted by the model because of parameter variability over time.

The full dataset extends from 1996 to 2020. We took a subset of the data from 1996 to 2008, computed Lotka-Volterra fits, and then compared those fits to the original data. The parameters are either displaying variation over time (Figs. 3, 4, 10, 11); or else, as evidenced by the poor R-squares for those subset fits, there is insufficient data from 1996 to 2008 to fit the Lotka-Volterra model. We can hope that parameter variability mitigates

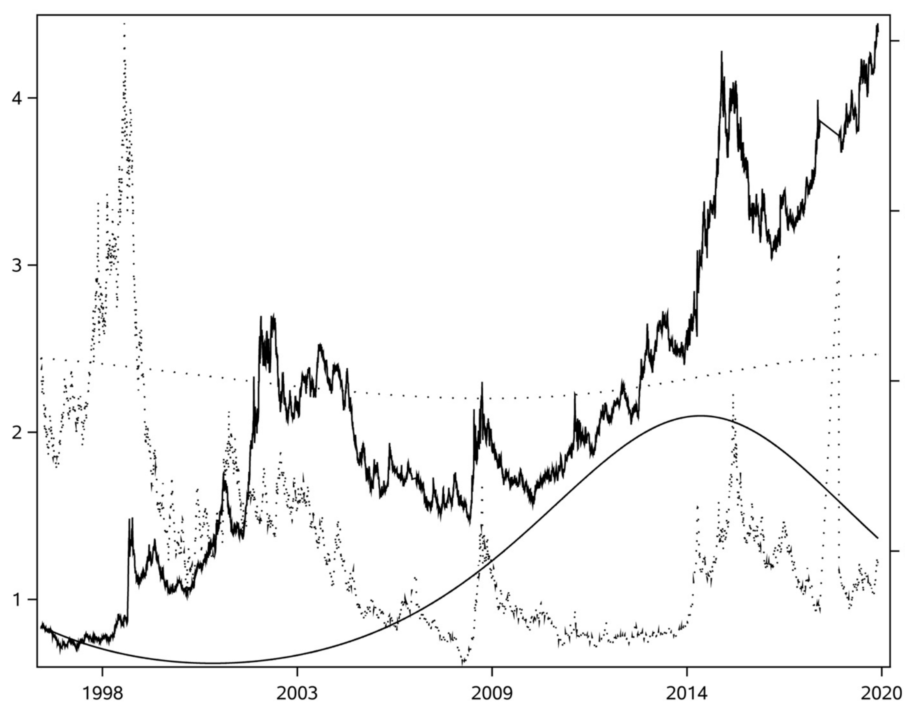


**Fig. 4.** BRL XCB time series plot showing parameter stability. This is a poor fit (Table 4). Solid lines are the currency and dotted lines are the commodity. Compare to Fig. 2. Dynamic Lotka-Volterra parameters were obtained using only the first half of the data and then used for the dynamic fit. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

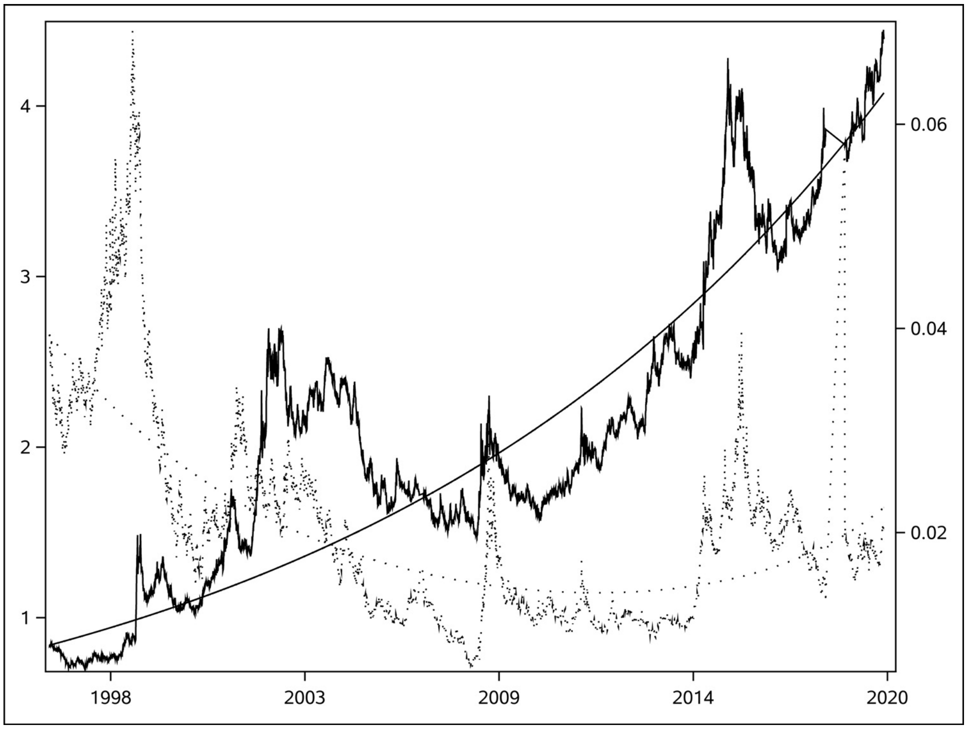


**Fig. 5.** BRL XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Jittery lines show the static solution and smooth lines show the dynamic solution. Parameter values and R-squares are shown in Tables 3 and 4. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.


**Fig. 6.** BRL XPT time series plot. Solid lines are the currency and dotted lines are the commodity. Jittery lines show the static solution and smooth lines show the dynamic solution. Parameter values and R-squares are shown in Tables 3 and 4. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.


**Fig. 7.** BRL XAU time series plot. Solid lines are the currency and dotted lines are the commodity. Jittery lines show the static solution and smooth lines show the dynamic solution. Parameter values and R-squares are shown in Tables 3 and 4. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

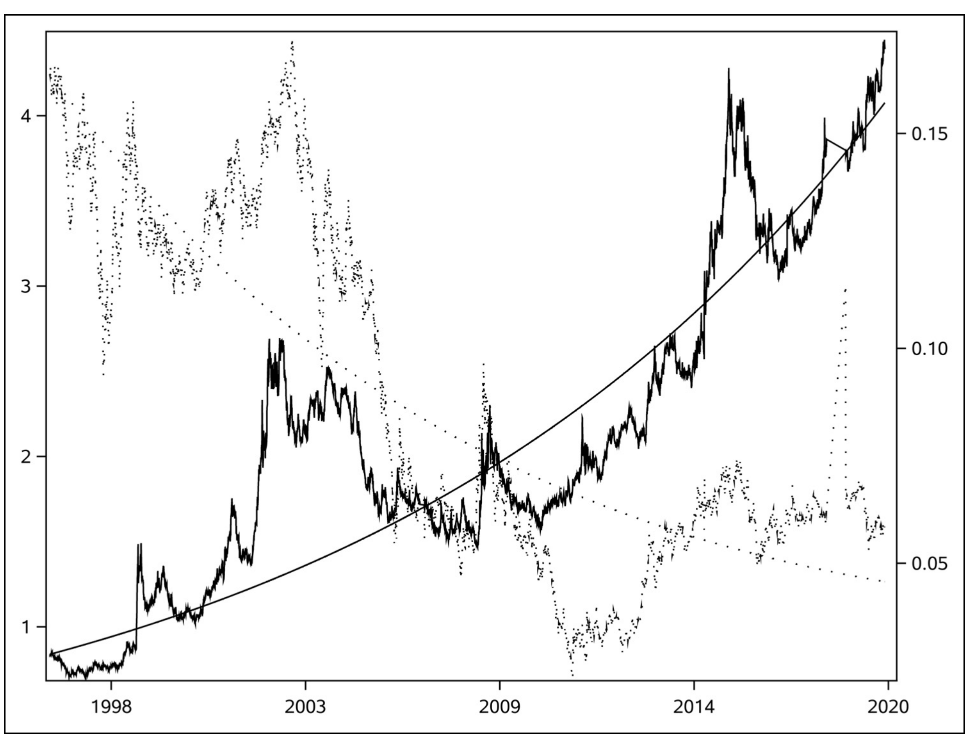


**Fig. 8.** BRL XAG time series plot. Solid lines are the currency and dotted lines are the commodity. Jittery lines show the static solution and smooth lines show the dynamic solution. Parameter values and R-squares are shown in Tables 3 and 4. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

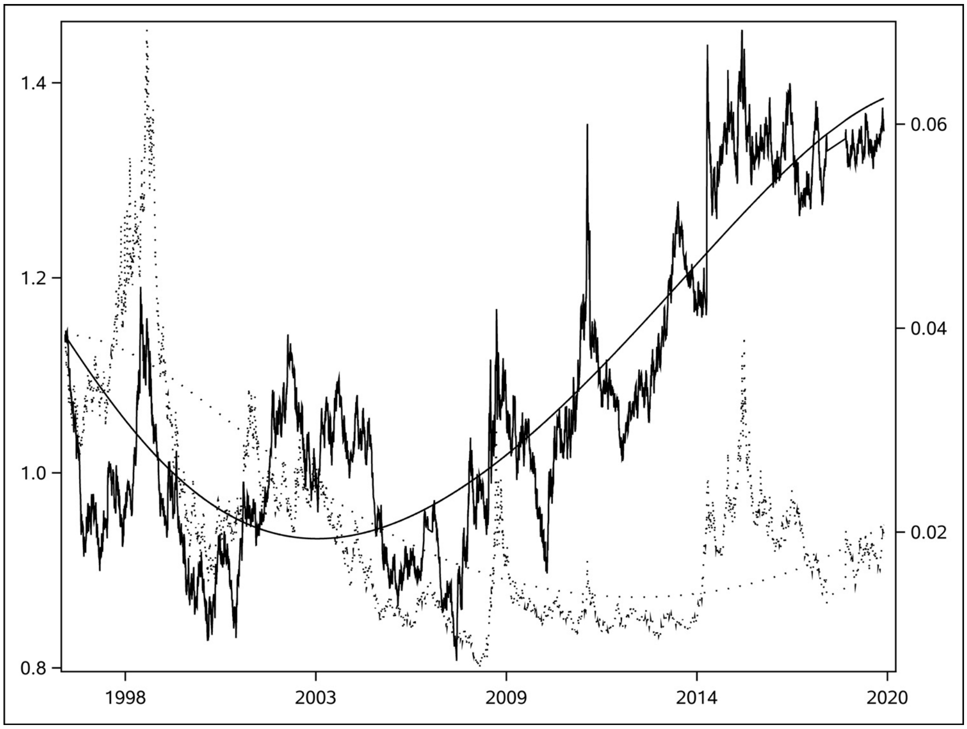


**Fig. 9.** CAD XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Jittery lines show the static solution and smooth lines show the dynamic solution. Parameter values and R-squares are shown in Tables 3 and 4. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

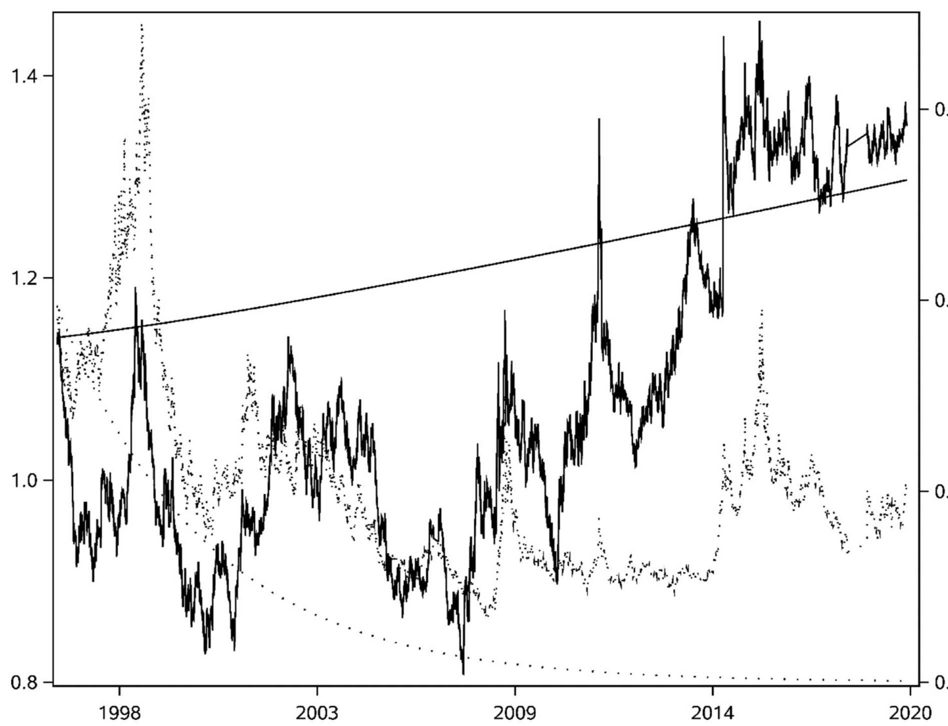


**Fig. 10.** CAD XCT time series plot showing parameter stability. Solid lines are the currency and dotted lines are the commodity. Compare to Fig. 9. Static Lotka-Volterra parameters were obtained using only the first half of the data and then used for the dynamic fit. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

hyperinflation, but we do not have evidence that the economy is a self-regulating goal-seeking system.

**Case 3.** We will see commodity-currency hyperinflation in the future.

The analyses in this paper demonstrate the existence of nonlinear coupling between commodities and currencies. Carried out over time, this coupling produces ultra-long hyperinflation waves. Unless Case 2 is correct and we can depend on parameter variability to mitigate hyperinflation, there is reason to expect cyclical hyperinflation.

*Damping Ultra-long Hyperinflation Waves*

Managing these commodity-currency systems will be challenging because they are nonlinearly coupled. However, the situation could

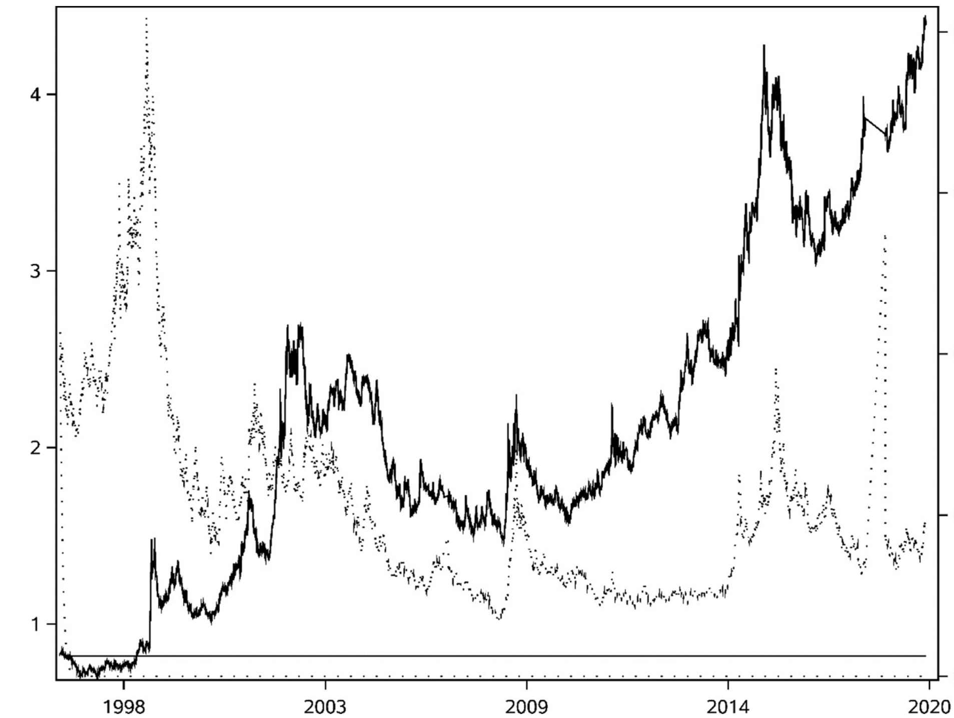


**Fig. 11.** CAD XCT time series plot showing parameter stability. This is a poor fit. Solid lines are the currency and dotted lines are the commodity. Compare to Fig. 9. Dynamic Lotka-Volterra parameters were obtained using only the first half of the data and then used for the dynamic fit. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

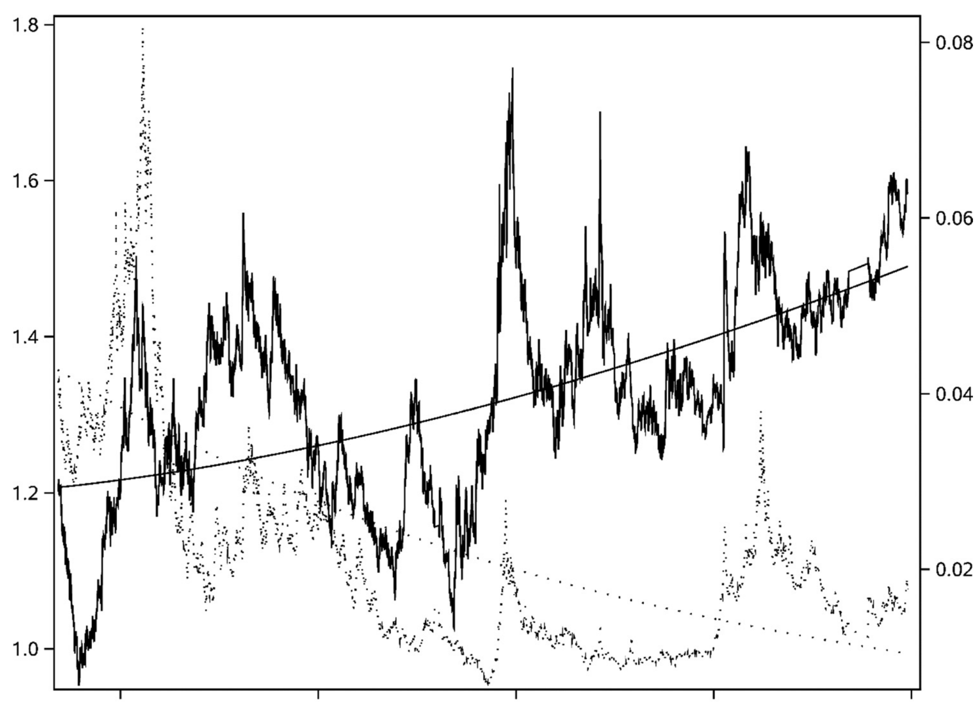


**Fig. 12.** NZD XCB time series plot. Solid lines are the currency and dotted lines are the commodity. Jittery lines show the static solution and smooth lines show the dynamic solution. Parameter values and R-squares are shown in Tables 3 and 4. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

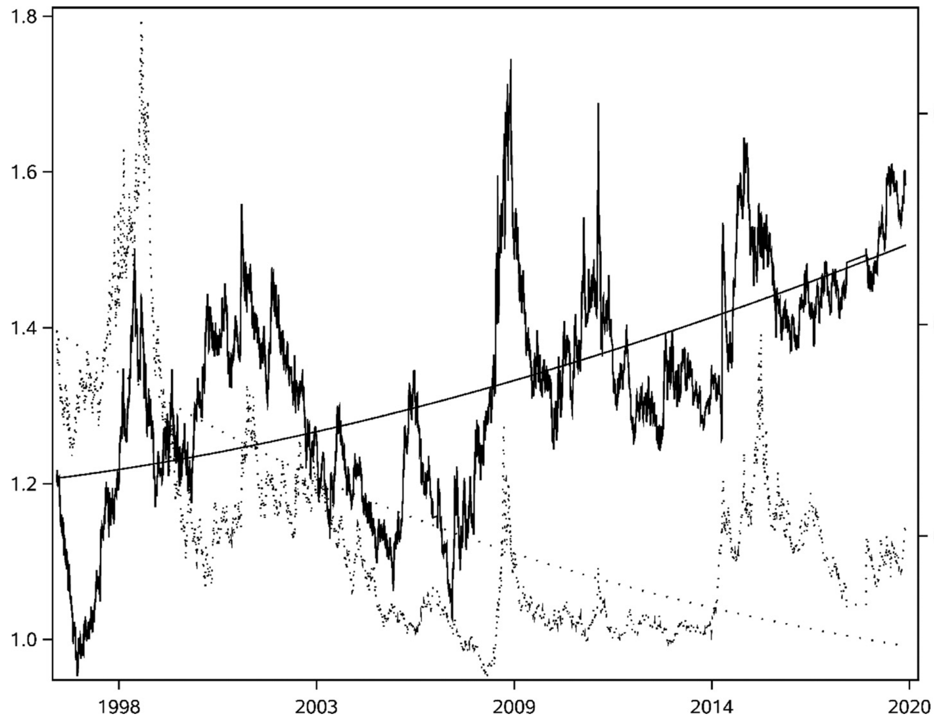


**Fig. 13.** NZD XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Jittery lines show the static solution and smooth lines show the dynamic solution. Parameter values and R-squares are shown in Tables 3 and 4. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

be much worse: because only two-dimensional model fits were achieved, there are only periodic solutions; there are no limit cycles, quasiperiodic solutions, or chaotic solutions. The lack of limit cycles is particularly important and promising. That means the potentially unsustainable oscillations are not attractors, to which a perturbed oscillation would return. Rather, if a commodity-currency system is perturbed into a new state, it will remain in that new state. That is a relatively simple case.

Based on our simulations, to transition to a lower amplitude oscillation, it is necessary to decrease $\alpha$ (the inflation rate of the currency when the commodity is absent) and/or $\gamma$ (the inflation rate of the commodity in the presence of the currency), and/or increase $\beta$ (the effect of the commodity on the currency) and/or $\delta$ (the price decrease rate of the commodity when the currency is absent). Moreover, to transition to a lower period oscillation, it is necessary to increase $\alpha$,

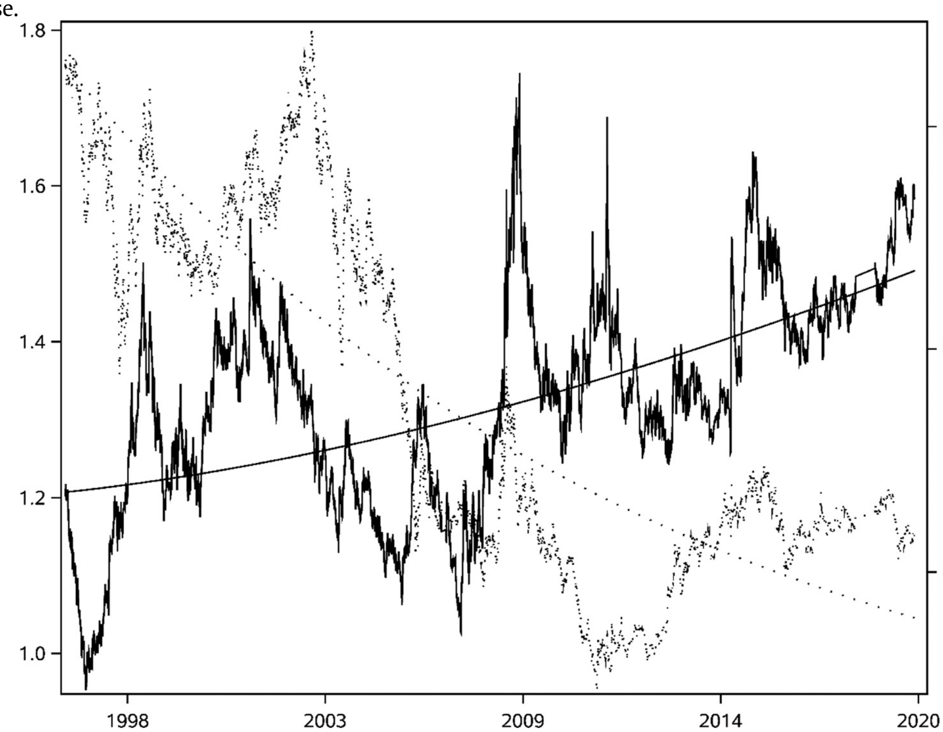


**Fig. 14.** NZD XAG time series plot. Solid lines are the currency and dotted lines are the commodity. Jittery lines show the static solution and smooth lines show the dynamic solution. Parameter values and R-squares are shown in Tables 3 and 4. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

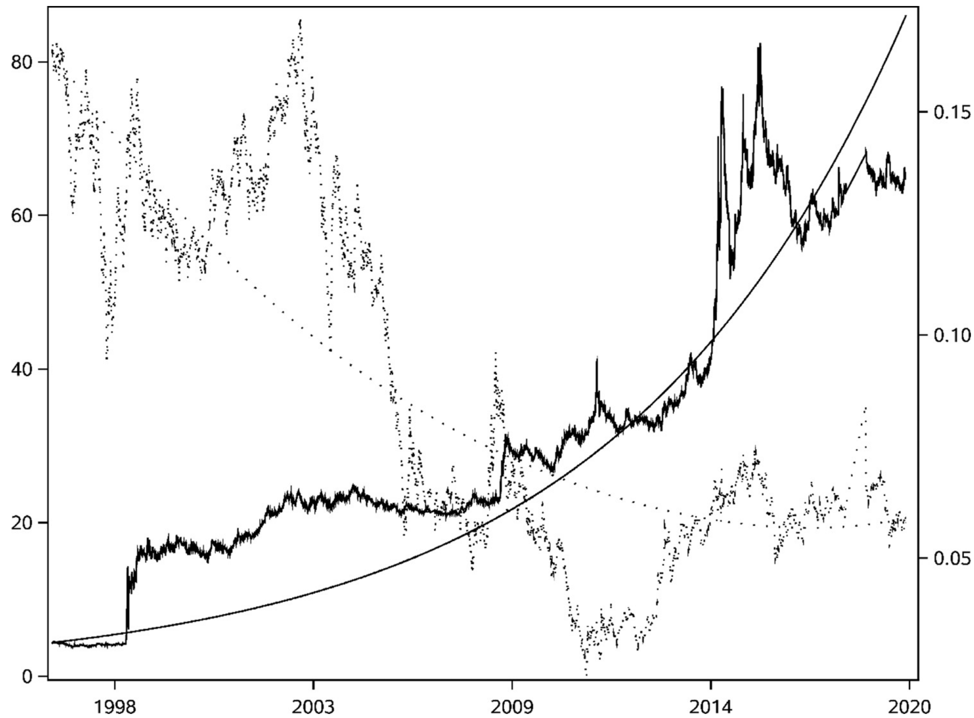


**Fig. 15.** RUB XAG time series plot. Solid lines are the currency and dotted lines are the commodity. Jittery lines show the static solution and smooth lines show the dynamic solution. Parameter values and R-squares are shown in Tables 3 and 4. Left y-axis is for currency. Right y-axis is for commodity. X-axis shows year.

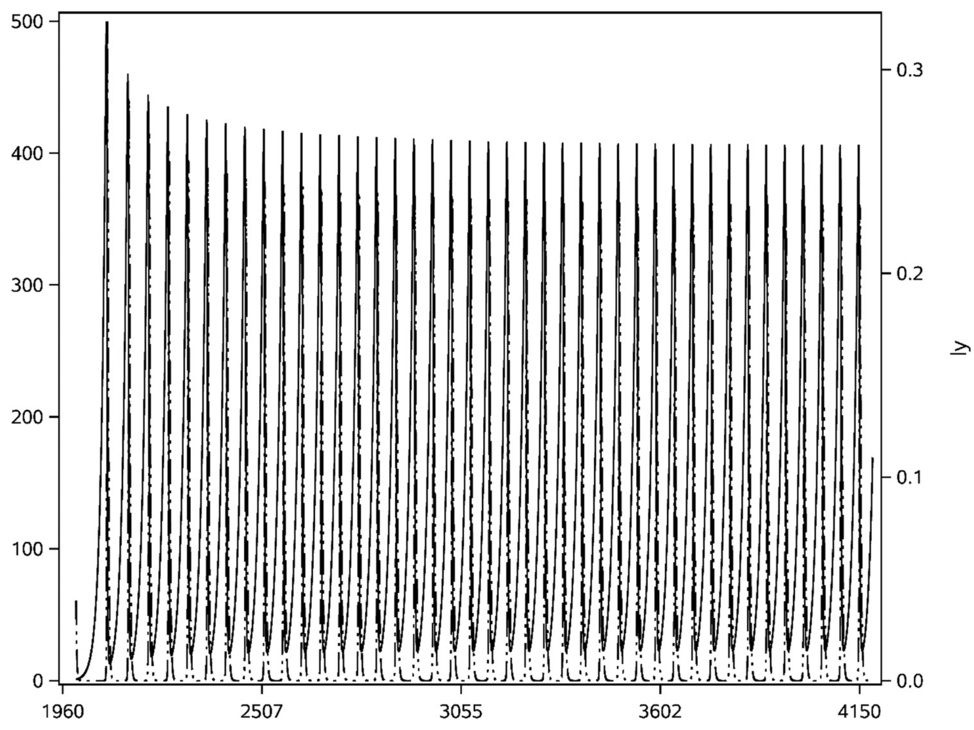


**Fig. 16.** CAD XCT simulated time series plot using the Lotka-Volterra parameters derived from the data. Solid lines are the currency and dotted lines are the commodity. Simulation using dynamic parameters computed from the data (Table 4). Solid line is currency (left y-axis), dotted line is commodity (right y-axis). Compare amplitudes to actual data in Fig. 7a. X-axis shows year.

$\beta$, and/or $\delta$ and/or decrease $\gamma$. To decrease both amplitude and period, it is necessary to increase $\beta$ and/or $\delta$ and/or decrease $\gamma$.

*Risk management*

Commodity-currencies have well established risks around commodity price shocks, economic sanctions, and trade policy (Giovannini et al., 2019). On the other hand, some commodities (particularly gold) are viewed by investors as 'safe haven' investments against both currency inflation and economic downturns (Junttila, Pesonen, & Raatikainen, 2018). Significant research demonstrated the link between commodity prices and inflation (Furlong & Ingenito, 1996, Cecchetti & Moessner, 2008), but commodity-currency hyperinflation is a risk that remains unaccounted for in contemporary literature.

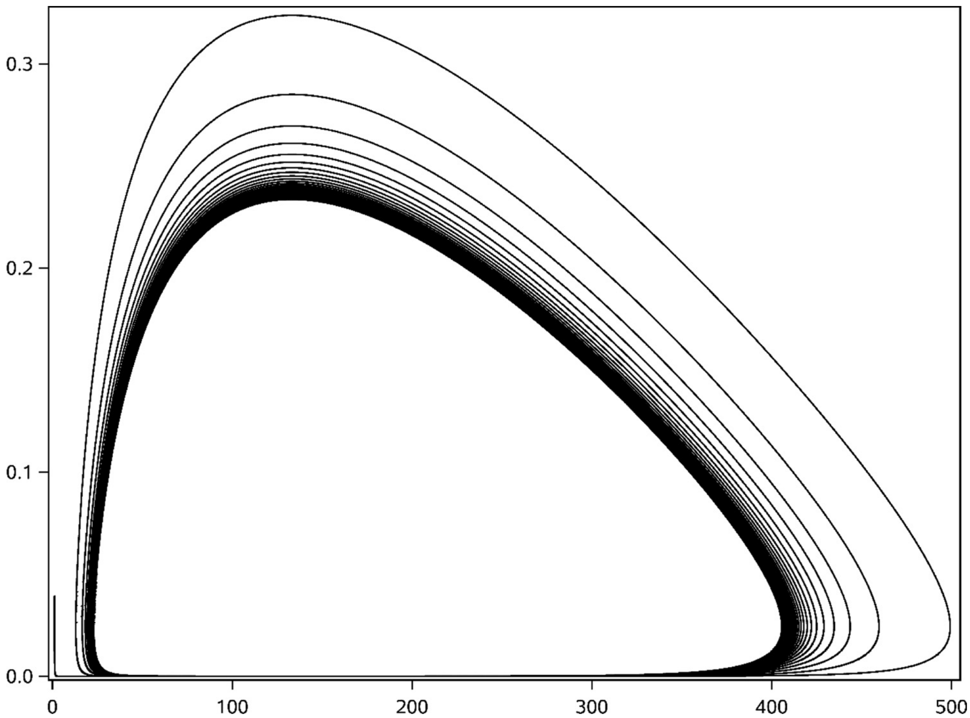


**Fig. 17.** Phase portrait of Fig. 16.

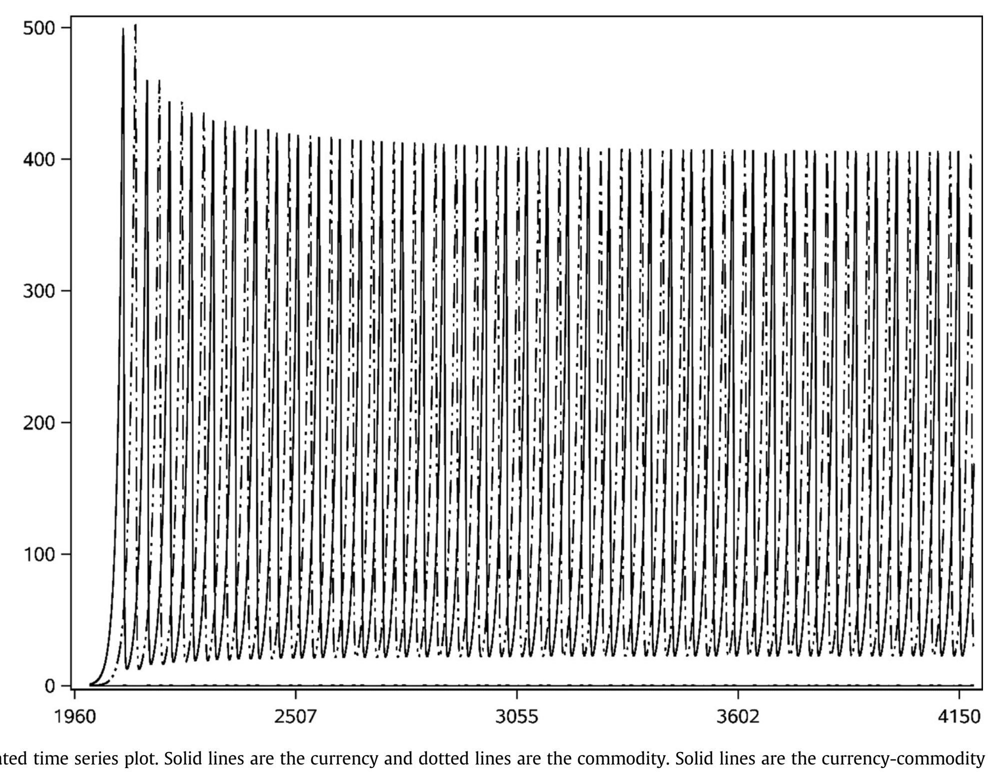


**Fig. 18.** CAD XCT simulated time series plot. Solid lines are the currency and dotted lines are the commodity. Solid lines are the currency-commodity pair with the actual initial value for CAD (x(0)=1.1409), dotted lines are the currency-commodity pair with x(0)=0.11409 (decreased by order of magnitude). Simulation using dynamic parameters computed from the data (Table 4). X-axis shows year.

Identifying the potential for hyperinflation in commodity-currencies has significant implications for finance, where the current notional value of risk-managing derivatives is estimated to be between 700 trillion and 1.5 quadrillion dollars (Krzyzanowski & Magdziarz, 2021). Most of these contracts are short term, where ample liquidity fuels demand and fairer risk pricing. Long-dated derivatives with maturities between one and ten years are less commonly traded and more difficult to price effectively. Further, even advanced applications of density forecasting utilizing oil futures fail to perform effectively beyond five years in the future (Bai, Li, Wei & Wei, 2020). Hyperinflation presents a significant new risk to the commodity-currency paradigm.

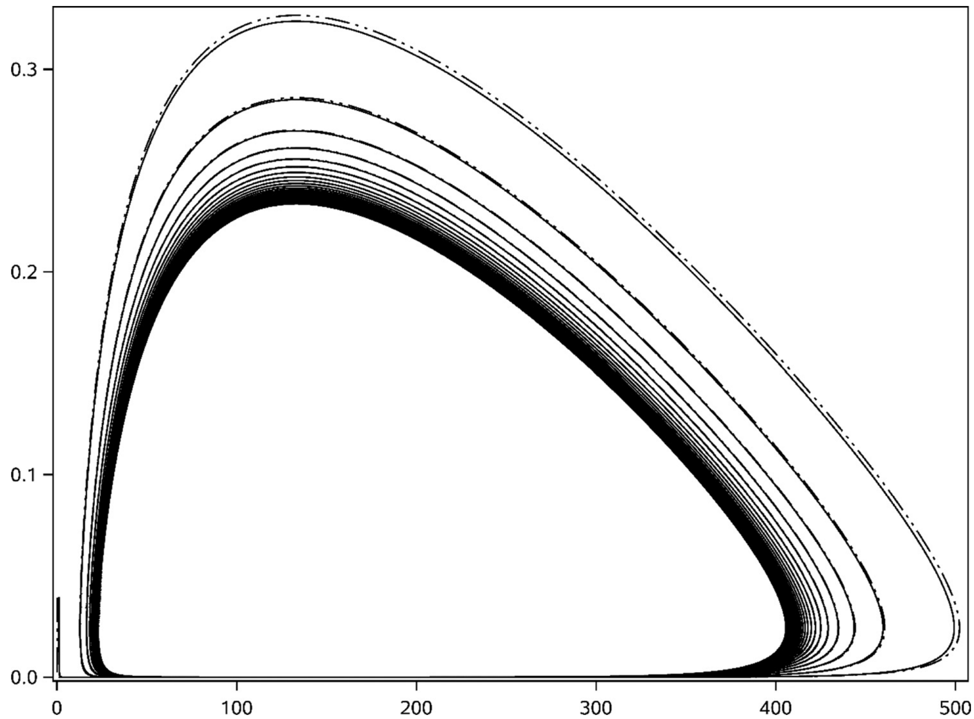


**Fig. 19.** Phase portrait of Fig. 18.

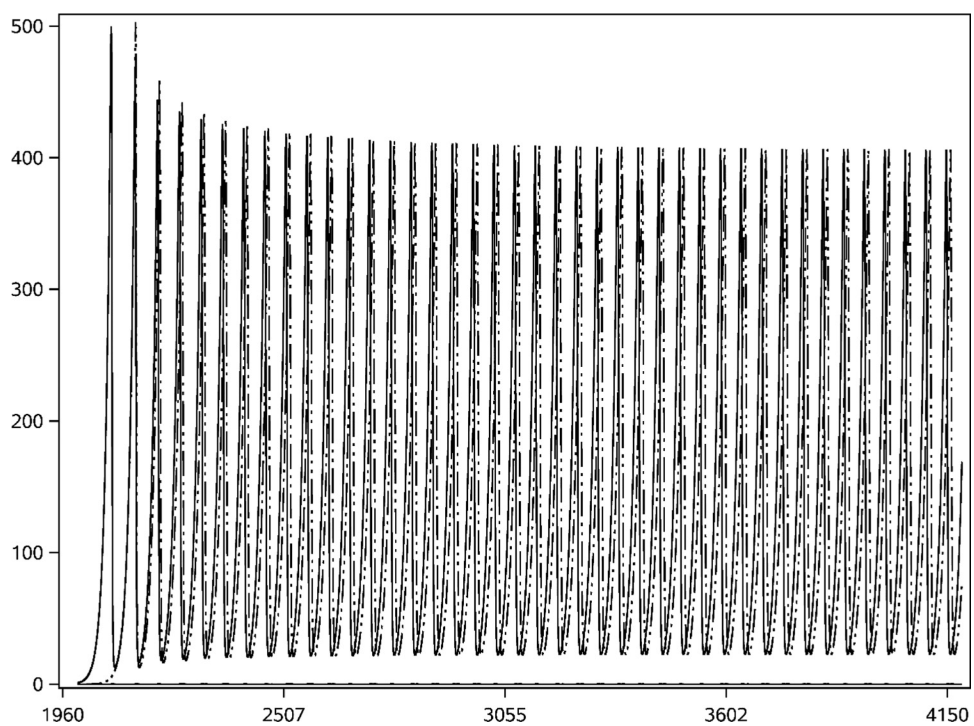


**Fig. 20.** CAD XCT simulated time series plot. Solid lines are the currency and dotted lines are the commodity. Solid lines are the currency-commodity pair with the actual initial value for CAD (x(0)=1.1409), dotted lines are the currency-commodity pair with x(0)=0.011409 (decreased by order of magnitude). Simulation using dynamic parameters computed from the data (Table 4). X-axis shows year.

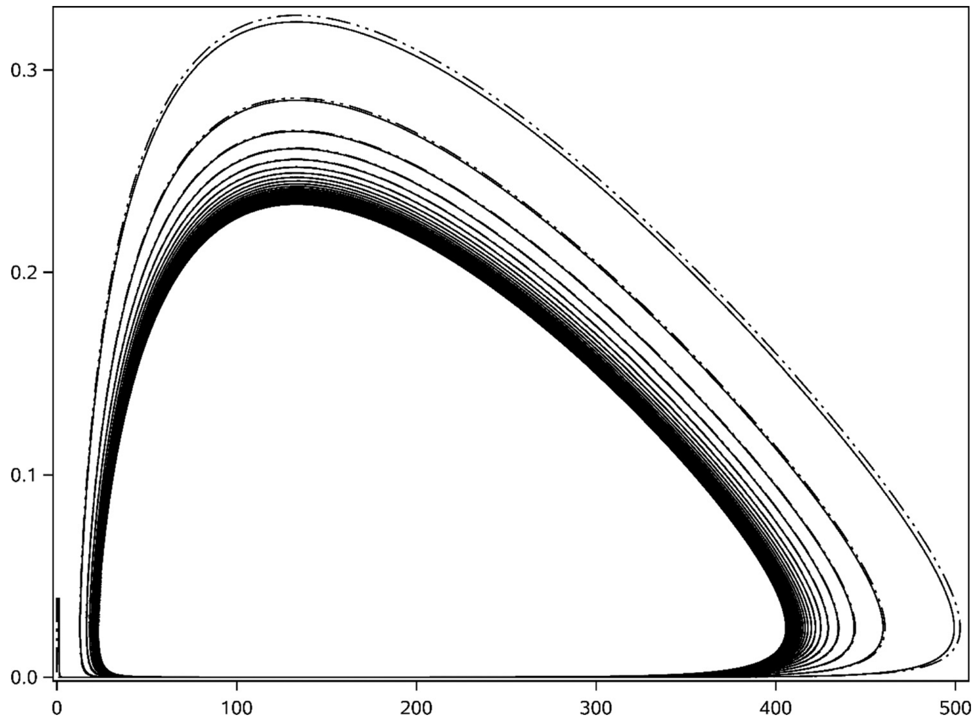


**Fig. 21.** Phase portrait of Fig. 20.

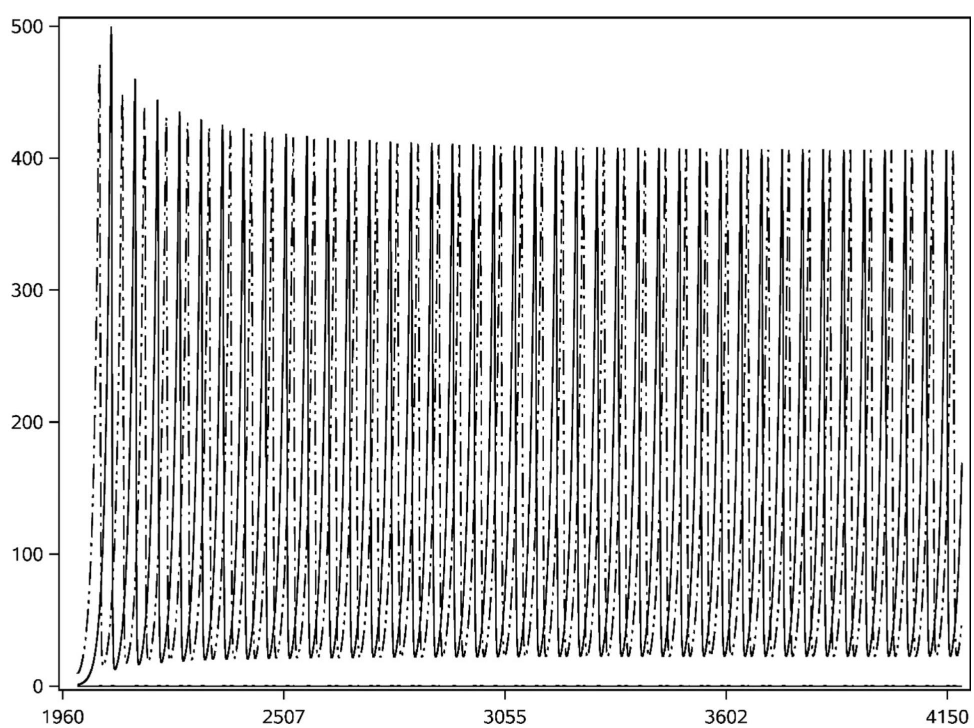


**Fig. 22.** AUD XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Solid lines are the currency-commodity pair with the actual initial value for CAD (x(0)=1.1409), dotted lines are the currency-commodity pair with x(0)=11.409 (decreased by order of magnitude). Simulation using dynamic parameters computed from the data (Table 4). X-axis shows year.

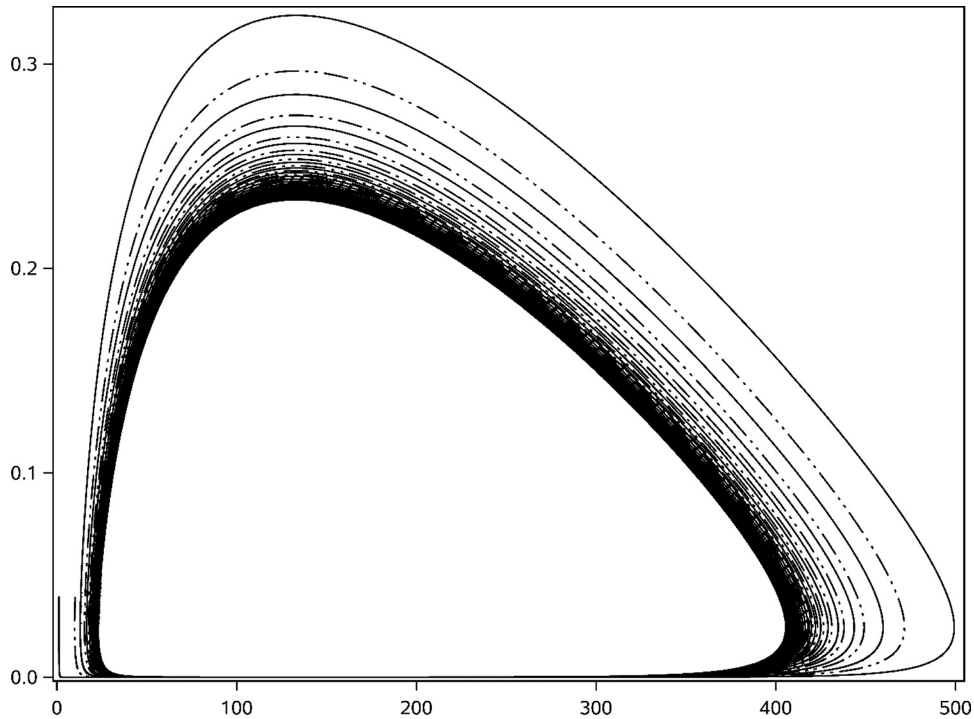


**Fig. 23.** Phase portrait of Fig. 22.

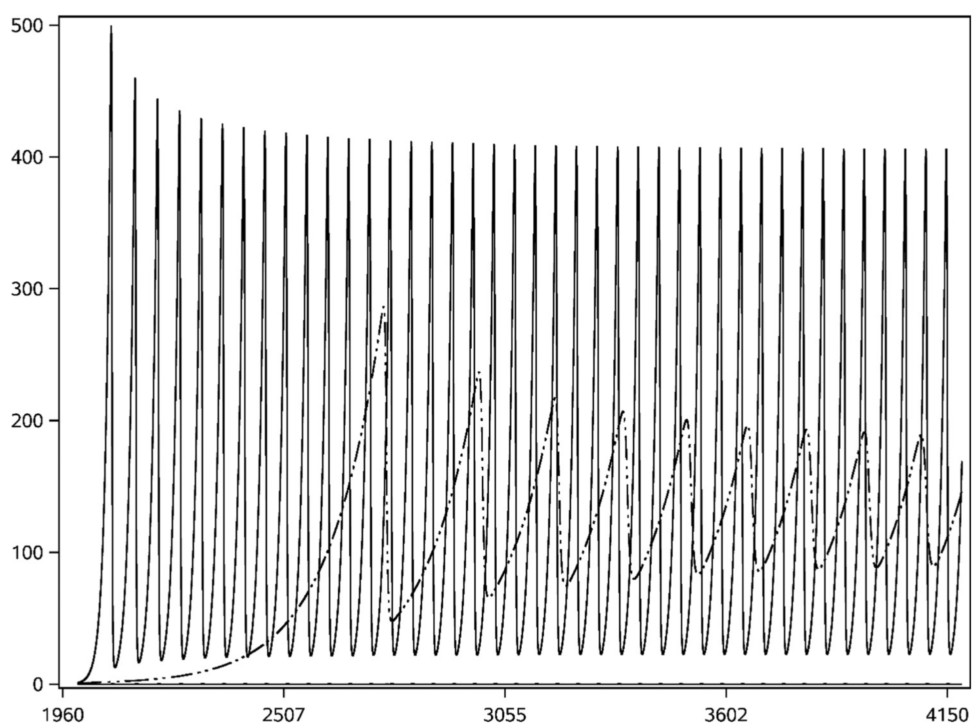


**Fig. 24.** CAD XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Solid lines are the currency-commodity pair with the actual Lotka-Volterra parameter value for $\alpha$ from the fit to the data ($\alpha$=0.000209), dotted lines are the currency-commodity pair with $\alpha$=0.0000209 (decreased by order of magnitude). Simulation otherwise using dynamic parameters computed from the data (Table 4). X-axis shows year.

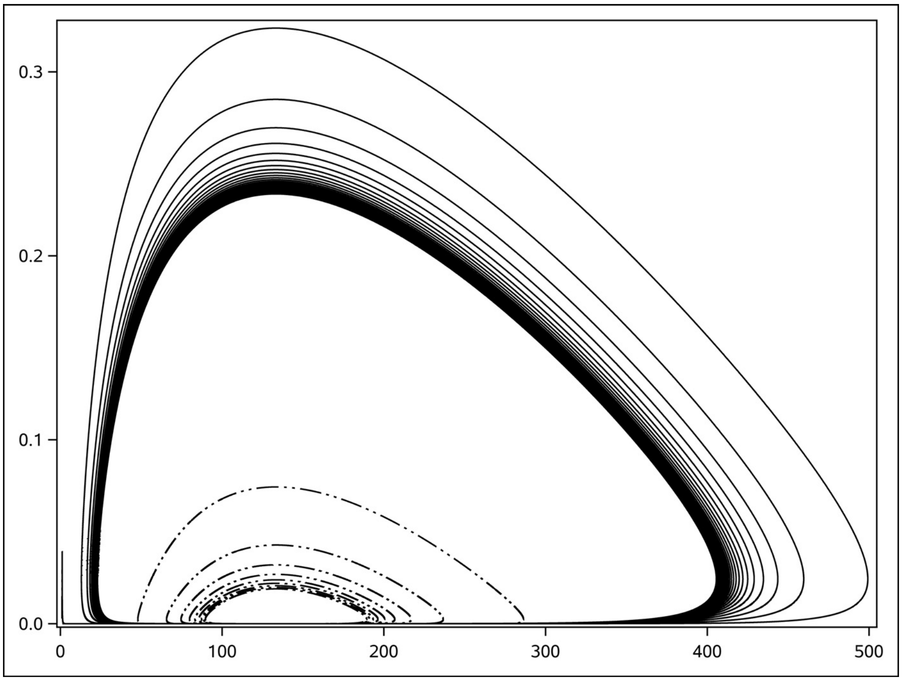


**Fig. 25.** Phase portrait of Fig. 24.

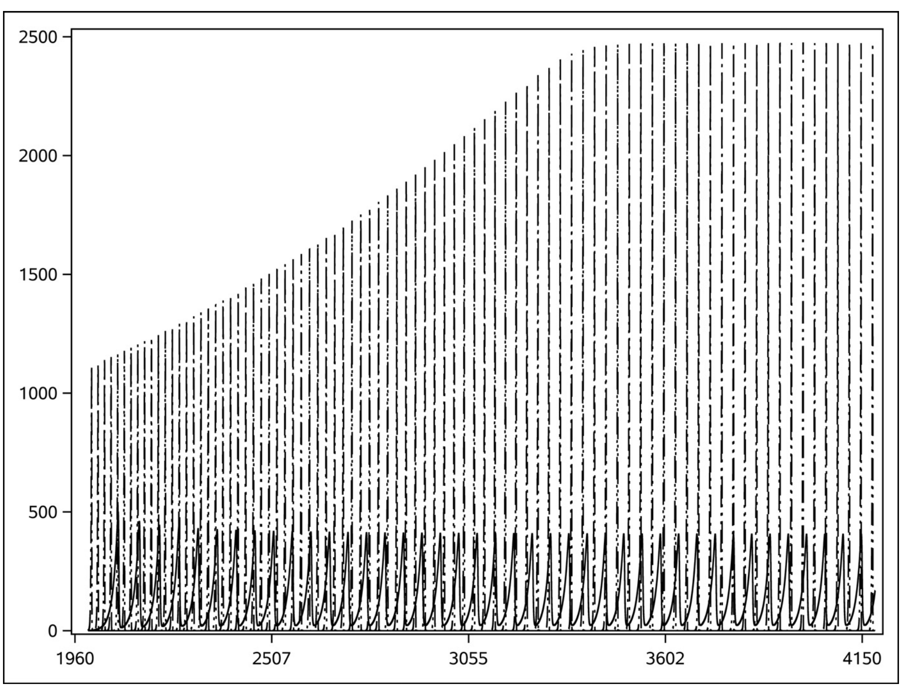


**Fig. 26.** CAD XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Solid lines are the currency-commodity pair with the actual Lotka-Volterra parameter value for $\alpha$ from the fit to the data ($\alpha$=0.000209), dotted lines are the currency-commodity pair with $\alpha$=0.00209 (increased by order of magnitude). Simulation otherwise using dynamic parameters computed from the data (Table 4). X-axis shows year.

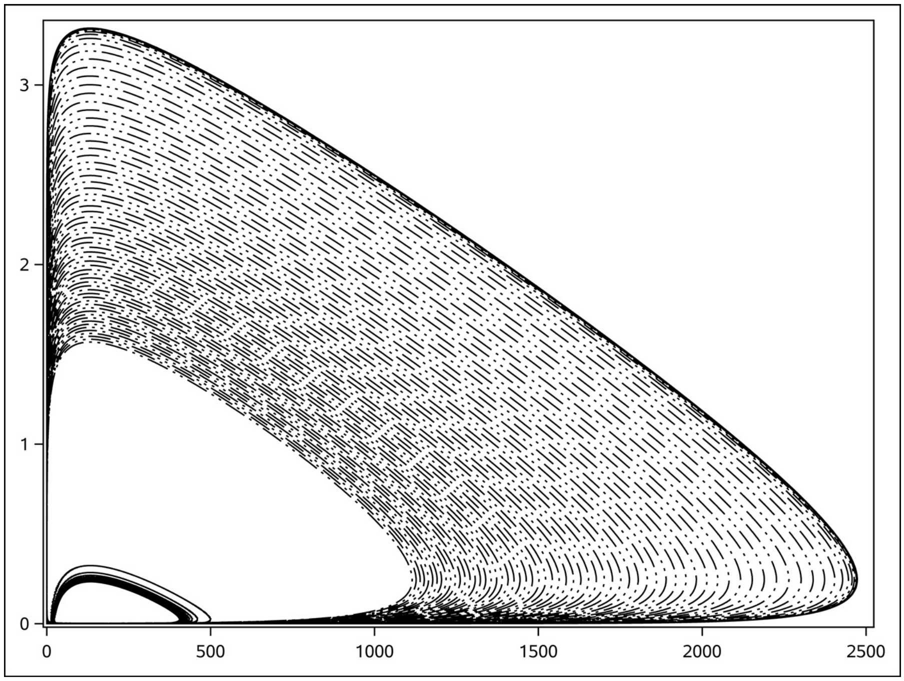


**Fig. 27.** Phase portrait of Fig. 26.

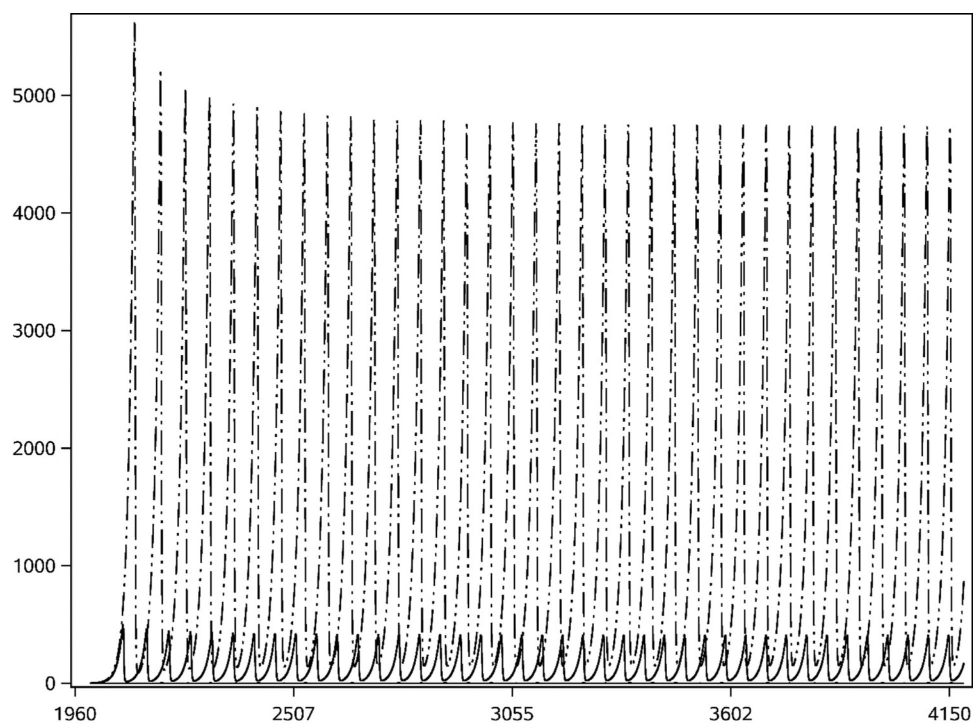


**Fig. 28.** CAD XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Solid lines are the currency-commodity pair with the actual Lotka-Volterra parameter value for $\beta$ from the fit to the data ($\beta$ =0.008611), dotted lines are the currency-commodity pair with $\beta$ =0.0008611 (decreased by order of magnitude). Simulation otherwise using dynamic parameters computed from the data (Table 4). X-axis shows year.

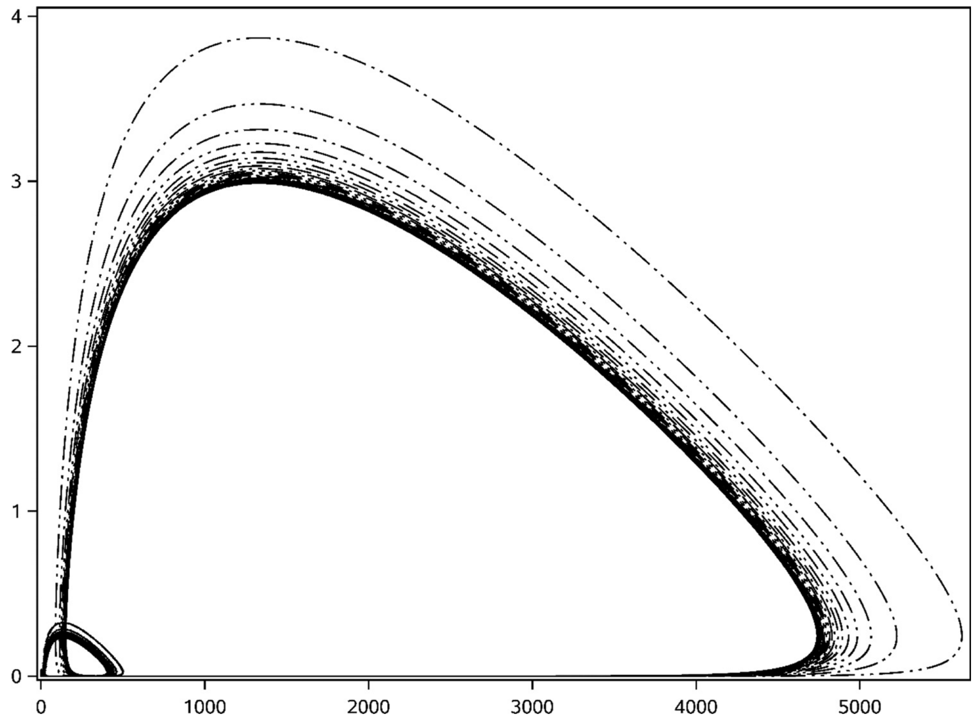


**Fig. 29.** Phase portrait of Fig. 28.

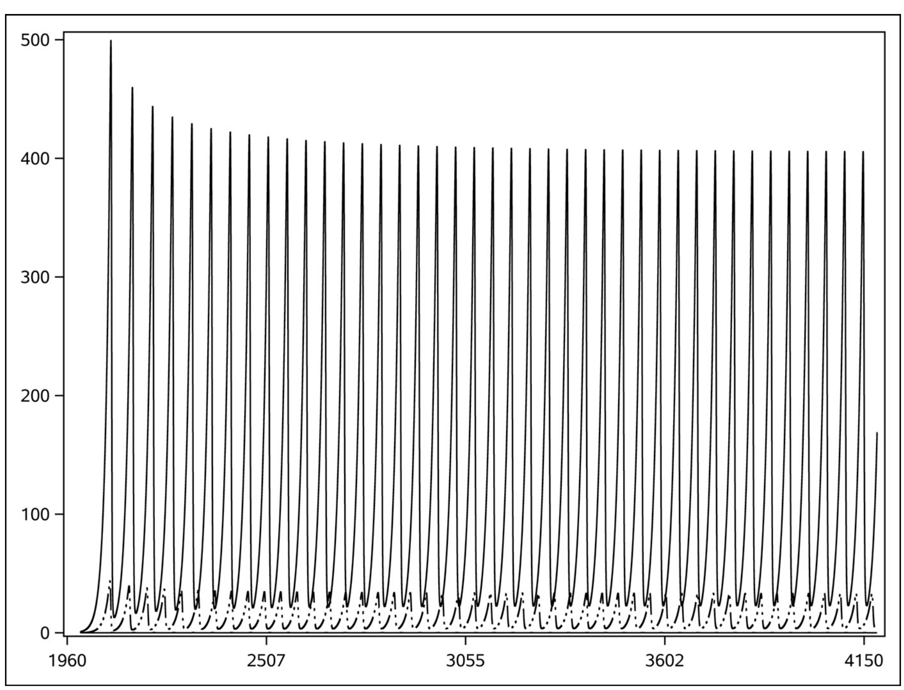


**Fig. 30.** CAD XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Solid lines are the currency-commodity pair with the actual Lotka-Volterra parameter value for $\beta$ from the fit to the data ($\beta$ =0.008611), dotted lines are the currency-commodity pair with $\beta$ =0.08611 (decreased by order of magnitude). Simulation otherwise using dynamic parameters computed from the data (Table 4). X-axis shows year.

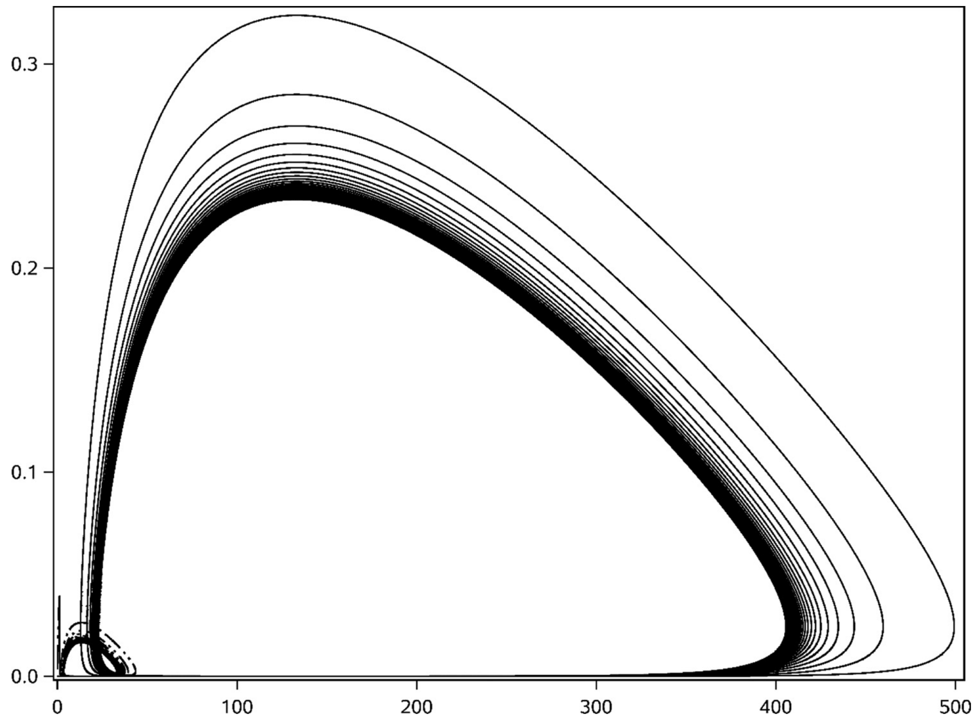


**Fig. 31.** Phase portrait of Fig. 30.

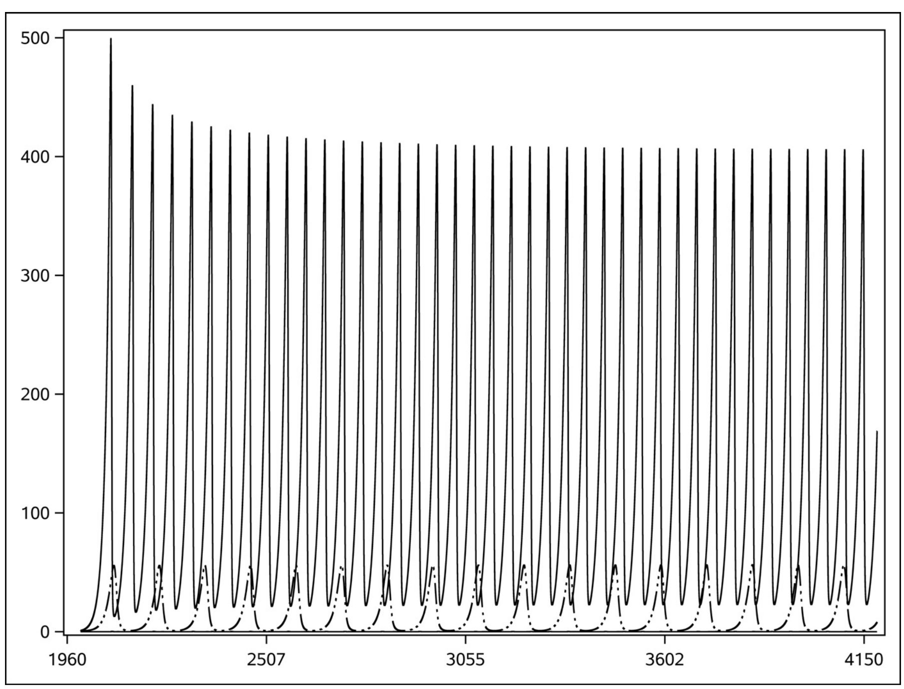


**Fig. 32.** CAD XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Solid lines are the currency-commodity pair with the actual Lotka-Volterra parameter value for $\gamma$ from the fit to the data ($\gamma$=0.001), dotted lines are the currency-commodity pair with $\gamma$=0.0001 (decreased by order of magnitude). Simulation otherwise using dynamic parameters computed from the data (Table 4). X-axis shows year.

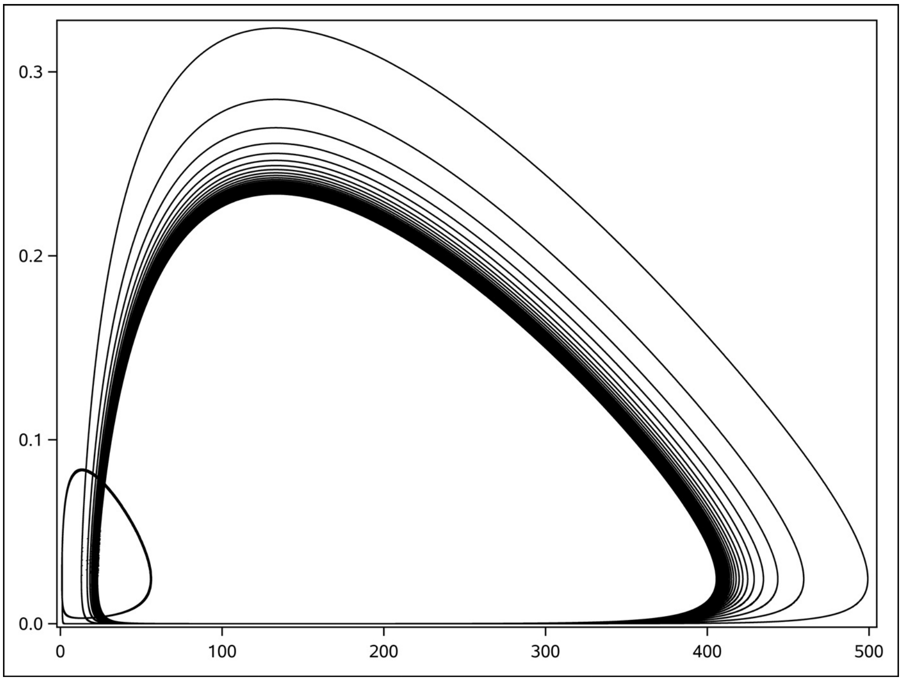


**Fig. 33.** Phase portrait of Fig. 32.

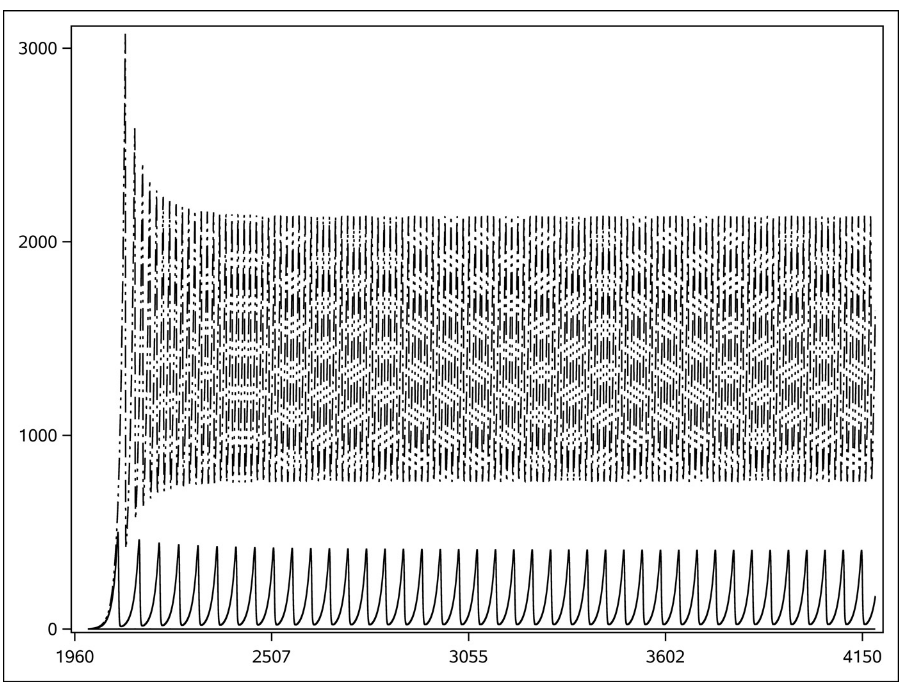


**Fig. 34.** CAD XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Solid lines are the currency-commodity pair with the actual Lotka-Volterra parameter value for $\gamma$ from the fit to the data ($\gamma$=0.001), dotted lines are the currency-commodity pair with $\gamma$=0.01 (increased by order of magnitude). Simulation otherwise using dynamic parameters computed from the data (Table 4). X-axis shows year.

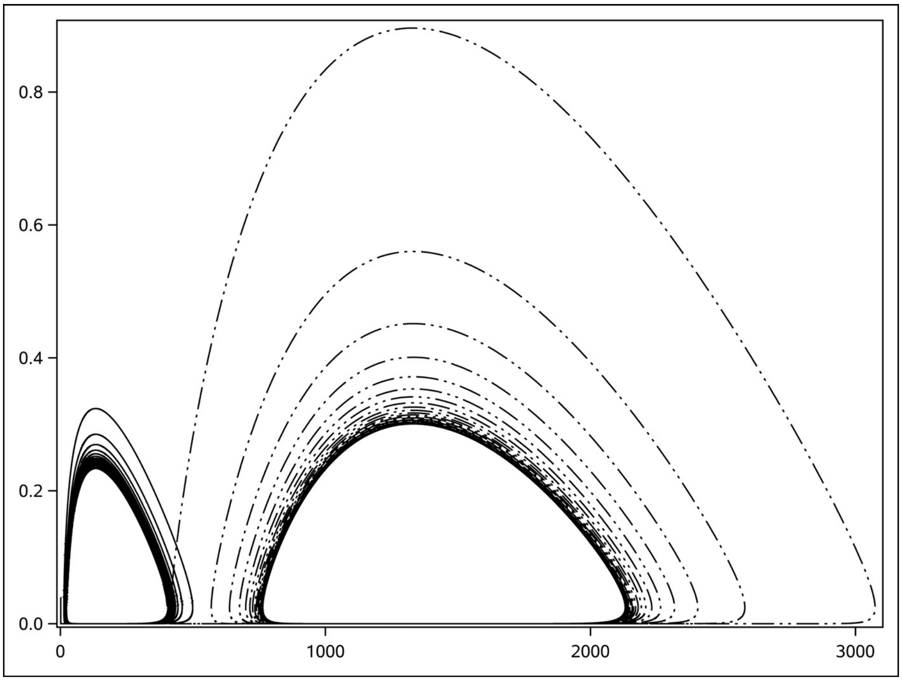


**Fig. 35.** Phase portrait of Fig. 34.

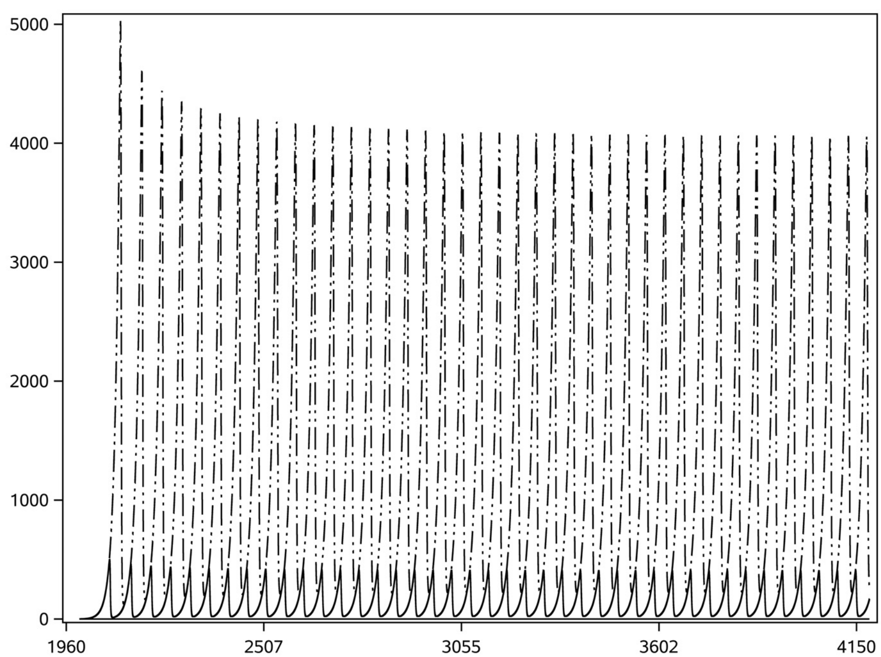


**Figure 36.** CAD XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Solid lines are the currency-commodity pair with the actual Lotka-Volterra parameter value for $\delta$ from the fit to the data ($\delta$=0.001), dotted lines are the currency-commodity pair with $\delta$=0.0001 (decreased by order of magnitude). Simulation otherwise using dynamic parameters computed from the data (Table 4). X-axis shows year.

High demand for sustainable technologies is putting greater demand on rare metals, where an estimated 11-to-26 fold increase in the supply of rare earth metals in needed to achieve global wind-power targets alone (Li et al., 2020). In addition to also being a potential extension to this research around commodity hyperinflation, this presents a significant barrier to an innovation-led sustainability transition.

## Conclusion

### *Theoretical implications*

We showed that select commodity-currency pairs can be modeled with second-order equations comprising endogenous oscillation-inducing feedbacks, with currencies leading commodities. These fits

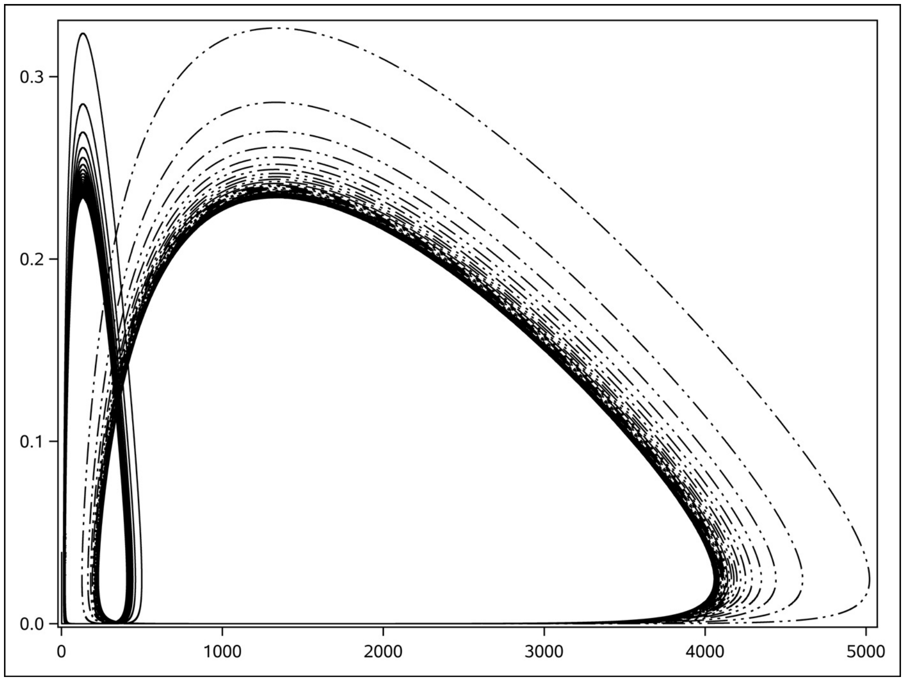


**Fig. 37.** Phase portrait of Fig. 36.

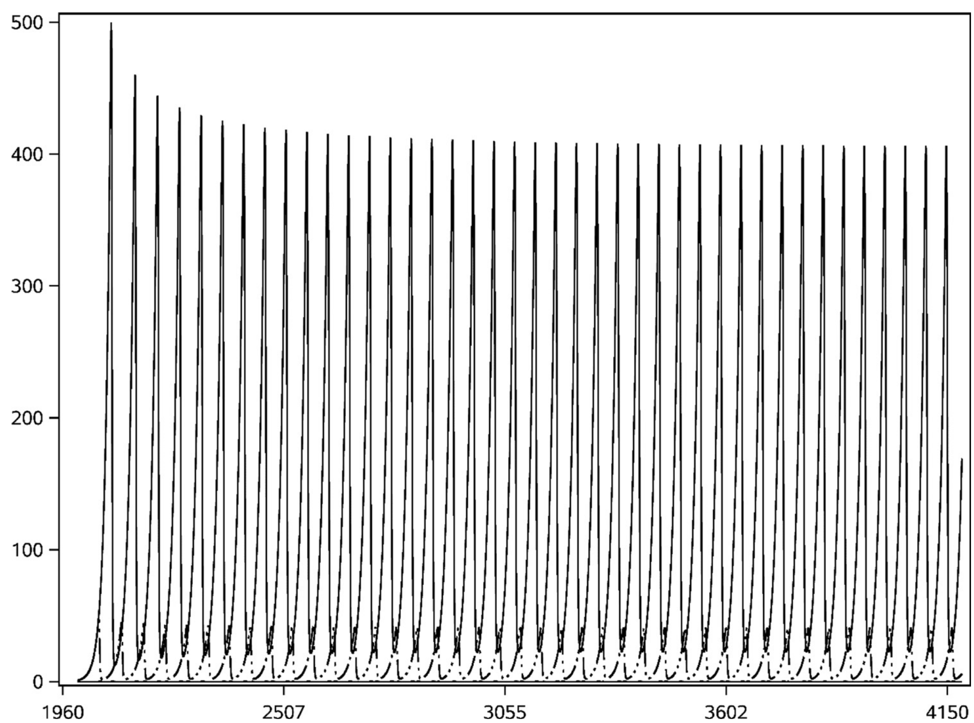


**Fig. 38.** CAD XCT time series plot. Solid lines are the currency and dotted lines are the commodity. Solid lines are the currency-commodity pair with the actual Lotka-Volterra parameter value for $\delta$ from the fit to the data ($\delta$=0.001), dotted lines are the currency-commodity pair with $\delta$=0.01 (increased by order of magnitude). Simulation otherwise using dynamic parameters computed from the data (Table 4). X-axis shows year.

implied the potential for ultra-long waves of unsustainable hyperinflation. These waves are much longer than Kondratiev waves and are a matter of first impression in the literature.

*Managerial implications*

We showed that we were able to decrease the amplitude and period of the potential oscillations by manipulating the numerical values of the equation coefficients. These results are meaningful because the coefficients *a priori* had real-world interpretations. Thus the present study gives us indications of how to decrease the potential for real-world ultra-long hyperinflation waves.

Amplitude in the Lotka-Volterra equations is said to depend on the initial values of the data, but our simulations show that the amplitudes of ultra-long waves depend on the model parameter values. That means devaluing or inflating the currencies or commodities is not a solution to hyperinflation. The solution to hyperinflation is to increase $\beta$ (the effect of the commodity on the currency) and/or $\delta$

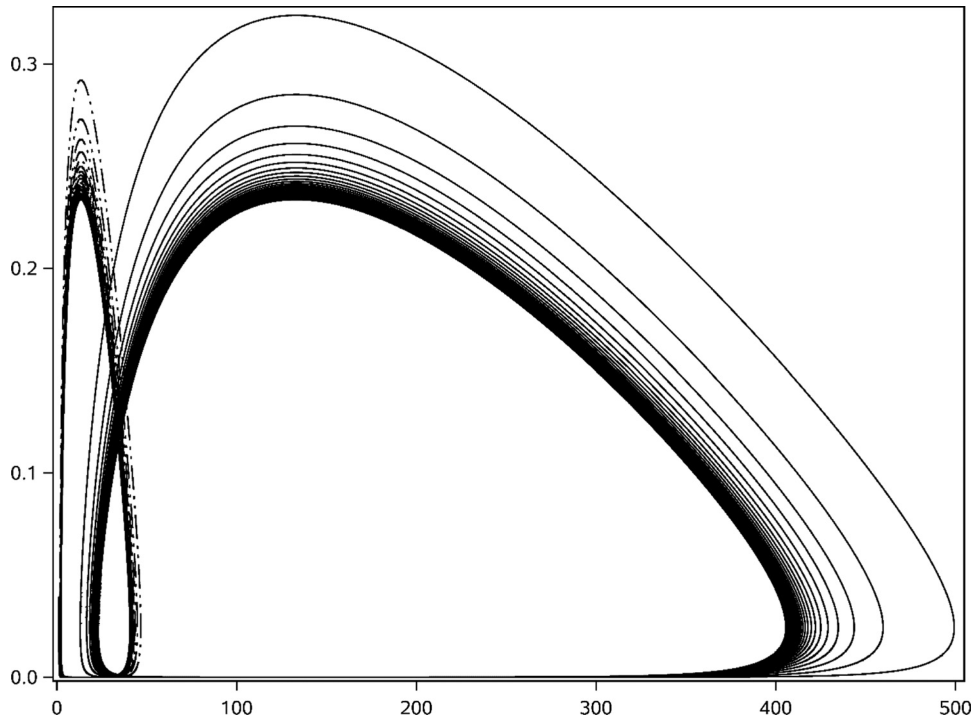


**Fig. 39.** Phase portrait of Fig. 38.

(the price decrease rate of the commodity when the currency is absent) and/or decrease $\gamma$ (the inflation rate of the commodity in the presence of the currency).

More broadly, this study shows the unintended consequences – hyperinflation - of a technological system designed to efficiently develop non-renewable resources. Entrepreneurial and corporate traders using such a system will have little to no control over the onset of hyperinflation. Neither will they be able to manage it.

*Limitations and future research*

We identified the solution to hyperinflation is to increase the effect of the commodity on the currency, and/or increase the price decrease rate of the commodity when the currency is absent, and/or decrease $\gamma$ (the inflation rate of the commodity in the presence of the currency. We did not discuss how this can be done in an actual economy. Future research may be directed towards remedying this shortcoming.

Commodities values have been found to lead currency exchange rates, using quarterly data. Using daily data, we were able to fit the two-dimensional Lotka-Volterra equations only to a subset of the commodity-currency pairs and only with currencies leading commodities (Table 2; Figs. 1-15). These differences bear further investigation.

We identified a strategy to treat hyperinflation: to decrease both amplitude and period, one must increase $\beta$ (the effect of the commodity on the currency) and/or $\delta$ (the price decrease rate of the commodity when the currency is absent) and/or decrease $\gamma$ (the inflation rate of the commodity in the presence of the currency). This strategy also bears further investigation, e.g., whether anything like it has ever been performed or what an intervention would look like.

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