A model for modifying the public transport service patterns to account for the imposed COVID-19 capacity

K. Gkiotsalitis

University of Twente, Center for Transport Studies, Horst – Ring Z-222, P.O. Box 217, 7500 AE Enschede, The Netherlands

ABSTRACT

As public transport operators try to resume their services, they have to operate under reduced capacities due to COVID-19. Because demand can exceed capacity at different areas and across different times of the day, drivers have to refuse passenger boardings at specific stops to avoid overcrowding. Given the urgent need to develop decision support tools that can prevent the overcrowding of vehicles, this study introduces a dynamic integer nonlinear program to derive the optimal service patterns of individual vehicles that are ready to be dispatched. In addition to the objective of satisfying the imposed vehicle capacity due to COVID-19, the proposed service pattern model accounts for the waiting time of passengers. Our model is tested in a bus line connecting the University of Twente with its surrounding cities demonstrating the trade-off between the reduced in-vehicle crowding levels and the excessive waiting times of unserved passengers.

1. Introduction

After the start of the pandemic, one country after the other implemented so-called social distancing measures affecting public transport, schools, shops, working places, and various other sectors (Anderson et al., 2020; Lewnard and Lo, 2020). To adjust their operations, some public transport service providers permitted the use of public transport for essential travel only (e.g., California and several other states in the US, Asia, and Europe) (Rodríguez-Morales et al., 2020). Crowded public transport services are considered one of the virus transmission factors. Thus, several office workers are asked to work from home as much as possible to reduce the burden on public transport services and ensure service availability for essential workers and vulnerable user groups.

Typical pandemic-related measures taken by public transport service providers include the limitation of the service span (e.g., not offering night services), the cancellation of certain lines, and the closure of selected stations by generating new service patterns (see the survey papers of Gkiotsalitis and Cats (2020) and Tirachini and Cats (2020)). Namely, Transport for London (TfL) suspended the night tube service and closed 40 metro stations that do not interchange with other lines (TfL, 2020). Similarly, the Washington Metropolitan Area Transit Authority (WMATA) closed more than 20% of its metro stations, reduced its service frequencies by more than half, and limited the operations of the daily metro services until 9 pm (WMATA, 2020). Valencia in Spain has also seen a reduction in service provision

of up to 35% (UITP, 2020b). In the Netherlands, service frequencies have been reduced significantly and the capacity of vehicles is reduced by allowing a limited number of standees.

Implementing specific measures to ensure social distancing is the main concern of public transport operators in post-lockdown societies. Several operators receive specific instructions from government authorities to operate under a pandemic-imposed capacity that does not allow them to use all available space inside a vehicle. This pandemic-imposed capacity aims to maintain sufficient levels of physical distancing among travelers but, at the same time, this might lead to significant numbers of unserved passengers and route/frequency changes. A recent study by Krishnakumari and Cats (2020) in the Washington DC metro system showed that if passengers are evenly spaced across platforms, each operating train can carry only 18% of its nominal capacity when implementing a 1.5-meter distancing and 10% when implementing a 2-meter distancing. Gkiotsalitis and Cats (2021) showed that the average seated train occupancy in the Washington metro can be reduced to 50%, 30%, and 20% when implementing 1-meter, 1.5-meter, and 2-meter social distancing policies, respectively. UITP (2020a) also reported that to ensure necessary social distancing between 1 and 1.5-meters the transport capacity has to be reduced to 25%-35%, which would hardly allow accommodating travel demand.

As part of their exit strategies, public transport authorities and operators have to devise strategies to meet the pandemic-imposed capacity limits. As of now, service providers have made adjustments

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Nomenclature

Sets

S set of ordered stops of the service line related to the trip of vehicle n, $S = \langle 1, \ldots, s, \ldots, |S| \rangle$;

Indices

n vehicle that is about to be dispatched;

Parameters

u_s array declaring how many consecutive times has stop $s \in S$ been skipped when trip n arrives at that stop (note: if s was served by the preceding trip $n-1$, then $u_s = 0$. If not, then $u_s \geqslant 1$);

g the pandemic-imposed capacity limit of the vehicle that performs trip n;

p_{sy} number of passengers waiting at stop $s \in S$ and are willing to travel to stop y at the time instance trip n arrives at stop s;

λ_{sy} the average passenger arrival rate at stop s whose destination is stop y at the time vehicle n arrives at stop s (note: $\lambda_{sy} = 0, \forall 1 \leqslant y \leqslant s$);

M a relatively large positive number that penalizes the consecutive skipping of the same stop;

h the planned time headway between two consecutive trips of the service line;

Decision variables

x_s $x_s = 1$ if vehicle n will serve stop s and $x_s = 0$ otherwise.

Variables

γ_s the passenger load of vehicle n when traveling from stop s to stop $s+1$.

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to meet the pandemic capacity, but so far these adjustments are devised and implemented in an ad hoc manner (UITP, 2020b). Typical measures include changes in service patterns by closing specific stops of the network that lead to overcrowding and adjustments of service frequencies. To rectify this, in this study we propose a dynamic service pattern model that decides about the skipped stops of every vehicle that is about to be dispatched to operate a particular trip. Our dynamic model can suggest a different service pattern for each trip of a service line using up-to-date passenger demand information to determine which stops should be skipped. The dynamic decision of skipped stops has some favorable properties compared to offline strategies that close stops for the entire day of operations:

- first, we can serve all stops during off-peak periods where the passenger demand does not exceed the pandemic-imposed capacity limitations; thus, resulting in fewer stop closures;
- second, we can utilize real-time data to decide about the skipped stops based on the current level of operations and the expected passenger demand in the short future;
- third, instead of permanently closing specific stops, we can alternate the skipped stops from trip to trip reducing the inconvenience of passengers.

By doing this, we exploit our vehicle resources as much as possible since we devise trip-specific service patterns instead of resorting to permanent stop closures. An additional problem during peak hours is that a vehicle reaches its pandemic-imposed capacity limit after serving just a few stops. If this vehicle and subsequent vehicles of the service line serve the initial stops of the line and skip the remaining stops, then passengers at the skipped stops will not be served until the demand returns to its pre-peak levels. To balance this, we propose a dynamic service pattern model that determines different service patterns for subsequent vehicles to ensure that stops that were skipped by the preceding trip have a higher chance to be served by the following trip. Our dynamic model determines the service pattern of every vehicle that is about to be dispatched whilst taking into consideration the skipped stops by past trips. In terms of computational complexity, solving our model can return an optimal service pattern in near real-time for realistic public transport lines that serve up to 60 stops.

The remainder of this study is structured as follows: Section 2 provides a literature review of service patterns and stop-skipping models. Section 3 introduces our integer nonlinear service pattern model and examines its computational complexity. Section 4 provides the imple-

mentation of our model in a bus line that connects the University of Twente with its two surrounding cities using multiple passenger demand scenarios. Finally, Section 5 concludes our work and offers future research directions, including the possibility of combining our dynamic service pattern model with dynamic frequency setting models to deploy more vehicles at peak periods of the day.

2. Literature review

Devising the service pattern of a vehicle at the operational level (e.g., when it is about to be dispatched) requires to determine which stops of the line should be served and which should be skipped by that vehicle (see (Li et al., 1991; Lin et al., 1995; Eberlein, 1997; Fu et al., 2003; Gkiotsalitis, 2020)). Determining the service pattern for each vehicle in isolation reduces the problem complexity and, similarly to our study, several works resort to exhaustive search methods (brute-force) to solve the dynamic problem taking advantage of the relatively small scale of the problem since typical public transport lines operate less than 40 stops (Fu et al., 2003; Sun and Hickman, 2005; Gkiotsalitis and Cats, 2021a).

The dynamic service pattern problem, which can be seen as a dynamic stop-skipping problem where all skipped stops are determined by the time the vehicle is dispatched, is typically modeled as a nonlinear integer program including assumptions of random distributions of boardings and alightings (Sun and Hickman, 2005). Similar to our work, Fu et al. (2003) used an exhaustive search to determine the skipped stops of one trip at a time. Fu et al. (2003) considered the total waiting times of passengers, the in-vehicle time, and the total trip travel time as problem objectives. The potential benefit was tested with a simulation of route 7D in Waterloo, Canada.

Different model formulations, such as Liu et al. (2013), imposed stricter stop-skipping constraints so that if a trip skips one stop, its preceding and following trip should not skip any stops. Liu et al. (2013) resorted to the use of a genetic algorithm incorporating Monte Carlo simulations because of the complexity of the formulated mixed-integer nonlinear problem. Eberlein (1995) simplified the problem by modeling it as an integer nonlinear program with quadratic objective function and constraints enabling its analytic solution.

The dynamic service patterns problems can be solved deterministically with high accuracy because the decisions are made every time a vehicle is about to be dispatched and the up-to-date information regarding the expected passenger demand and the inter-station travel times results in low estimation errors of the realized demand and tra-

vel times. Several works, however, do not solve this problem dynamically and consider stochastic travel times and passenger demand in the problem formulation because they make decisions about future daily trips. Chen et al. (2015) used an artificial bee colony heuristic to solve the offline service pattern problem considering stochasticity since this work determined the service patterns of several vehicles ahead. Gkiotsalitis (2019) used also a robust optimization model for devising the service patterns of all daily trips considering the stochastic travel-times in the objective function and integrating them into a genetic algorithm that tried to find service patterns that perform reasonably at worst-case scenarios. Service patterns have also been derived for clusters of trips at the offline level (Verbas and Mahmassani, 2015; Verbas et al., 2015; Gkiotsalitis et al., 2019). The aforementioned works do not exploit the real-time information from telematics systems and automated passenger counts and are not in-line with the objectives of our work that needs to make well-informed decisions using up-to-date information to avoid vehicle overcrowding. Therefore, offline approaches that determine service patterns or combine service patterns with offline timetabling and offline vehicle scheduling (e.g., Li et al. (1991), Cao et al. (2016), Gao et al. (2016), Altazin et al. (2017), Cao and Ceder (2019)) will not be the main focus of this study.

In past works, stop-skipping has been combined with vehicle holding (see Eberlein (1995), Lin et al. (1995), Cortés et al. (2010), Sáez et al. (2012), Nesheli et al. (2015) or the recent work of Zhang et al. (2020)). However, such works have different objectives compared to the objectives of our model, such as improving the service regularity by implementing vehicle holding or reducing the in-vehicle travel times, which might be counterproductive when trying to maintain a pandemic-imposed capacity. For instance, when a vehicle is held at a stop more passengers will arrive at the stop and will be willing to board the vehicle leading to higher crowding levels).

From the current literature, we identify a specific research gap. Whereas there is an extensive body of works on service pattern models, these works focus on improving the in-vehicle travel times, the waiting times of passengers at stops, or the service regularity. To the best of the author's knowledge, there are no works, particularly at the dynamic level, that consider the capacity limitations of vehicles when devising service patterns. Hence, they do not account for the (potential) negative effect of overcrowding that increases the risk of COVID-19 transmission. This motivates our work which proposes a novel model formulation that explicitly considers the pandemic-imposed capacity limit as the main problem objective with the use of extensive penalties for vehicles that do not meet this limit when departing from a stop.

With the introduction of our model, our work addresses the following research questions:

(a) how can we improve the crowding levels inside vehicles to meet the pandemic-imposed capacity when changing the service patterns of vehicles?
(b) what are the side effects in terms of unserved passenger demand and increased passenger waiting times when applying such service patterns?

3. Service pattern model

3.1. Model formulation

Our service pattern model can be applied to a service line and it determines the service pattern of every vehicle n that is about to be dispatched. The service pattern decision is made before the vehicle is dispatched so as to inform the passengers waiting at stops about the skipped stops by that vehicle. Offline service pattern models result in stop closures that skip the same stops repeatedly. This, however, might reduce the accessibility of passengers that would like to use the permanently skipped stops as part of their origin–destination trip.

In our dynamic service pattern formulation, we take into consideration:

- the number of waiting passengers at each stop to evaluate the impact of serving a stop to the crowding levels of the vehicle;
- the skipped stops from previous trips of the line to prioritize stops that have not been served by previous trips.

It is important to note that when vehicle n is about to be dispatched, its preceding trips are already operating their services and their service patterns are known. That is, every time we determine the service pattern of a trip n, the service pattern of the previously dispatched vehicles is taken into consideration.

The modeling part of this work relies on the following assumptions:

(1) The passenger arrivals at stops are random because the passengers cannot coordinate their arrivals with the arrival times of buses at high-frequency services (Welding, 1957; Randall et al., 2007);
(2) A passenger will wait for the next trip of the same line if a vehicle skips his/her stop.
(3) Even if a stop is skipped, passengers can disembark at that stop because the vehicle will stop at every stop of the line for alighting passengers. That is, a skipped stop is a stop where passengers cannot board the vehicle.
(4) Overcrowding at the location of a public transport stop is not an issue because the stops are outdoors and the virus transmission risk is much lower compared to the transmission risk in indoor spaces (e.g., inside the vehicle).

The third assumption implies that each vehicle will allow passengers to disembark at any stop of the line, even at the "skipped stops" where new passengers are not allowed to board the vehicle. The last assumption implies that our service pattern model cannot be applied to public transport services with stops in indoor spaces (i.e., metro systems or long-distance rail services).

Before proceeding to the model formulation, we introduce the following nomenclature:

The service pattern model that determines which stops will be skipped by vehicle n is presented in Eqs. (1)–(6):

$$\min f(\mathbf{x}) := \sum_{s \in S} \sum_{y \in S | y > s} \frac{1}{2} \left[(u_s + (1 - x_s)) \cdot h \cdot p_{sy} + h^2 \cdot \lambda_{sy} \right]$$

$$+ M \sum_{s \in S} [u_s + (1 - x_s)]^2 \tag{1}$$

$$\text{s.t.} : \gamma_s \leqslant g \quad (\forall s \in S) \tag{2}$$

$$\gamma_1 = x_1 \sum_{y \in S} p_{1y} \tag{3}$$

$$\gamma_s = \gamma_{s-1} + x_s \sum_{y \in S | y > s} p_{sy} - \sum_{y \in S | y < s} p_{ys} x_y \quad (\forall s \in S \setminus \{1, |S|\}) \tag{4}$$

$$\sum_{s \in S \setminus \{|S|\}} x_s \geqslant 1 \tag{5}$$

$$x_s \in \{0, 1\} \quad (\forall s \in S) \tag{6}$$

The objective function (1) consists of two components. The first component,

$$\sum_{s \in S} \sum_{y \in S | y > s} \frac{1}{2} \left[(u_s + (1 - x_s)) \cdot h \cdot p_{sy} + h^2 \cdot \lambda_{sy} \right]$$

computes the expected waiting time of all passengers that wait at every stop $s \in S$ by the time the following trip $n + 1$ of our current trip n arrives at stop s. Our mathematical program strives to minimize this waiting time. In more detail,

$$\frac{1}{2} (u_s + (1 - x_s)) \cdot h \cdot p_{sy}$$

is the waiting time of all passengers that were waiting at stop s when vehicle n arrived at that stop. This waiting time is equal to the number of passengers at stop s that are not served by previous trips, p_{sy}, multi-

plied by $\frac{1}{2}(u_s + (1 - x_s)) \cdot h$. In the latter term, (i) h is the time headway between two successive trips of the line, (ii) u_s indicates the number of successive trips that skipped stop s since the last time it was served by a vehicle, and (iii) $1 - x_s$ indicates whether stop s will be served by the current trip n (if yes, $1 - x_s = 0$, if not, $1 - x_s = 1$).

The term $(u_s + (1 - x_s)) \cdot h$ is equal to $u_s \cdot h$ if the current trip n serves stop s. In this case, it indicates the time headway between the last time stop s was served by a trip of the line, and the time instance stop s is served by trip n. In the case that $x_s = 0$, we have $(u_s + (1 - x_s)) \cdot h = (u_s + 1) \cdot h$ because the waiting passengers at stop s will still have to wait for an additional time h until the next trip $n + 1$ arrives and we need to make a stop-skipping decision for it. Finally, we note that we divide the value $(u_s + (1 - x_s)) \cdot h \cdot p_{sy}$ by 2 because we assume that in high-frequency services passenger arrivals at stops are random and the average passenger from the waiting passengers $\langle 1, 2, \ldots, p_{sy} \rangle$ will have to wait half of the total waiting time $(u_s + (1 - x_s)) \cdot h$. Note that if $(u_s + (1 - x_s)) \cdot h$ was not divided by 2, we would have assumed that all p_{sy} passengers arrived at stop s immediately after the departure of the last trip that served stop s, and this is unrealistic. In summary:

- if trip n serves stop s, the p_{sy} passengers that waited at stop s when trip n arrived would have been waiting for $\frac{1}{2} u_s \cdot h$ until they finally board;
- if trip n does not serve stop s, our decision will force the p_{sy} passengers to wait at stop s for one more headway $h : \frac{1}{2}(u_s + (1 - x_s)) \cdot h = \frac{1}{2}(u_s + 1) \cdot h$.

In addition,

$$\frac{1}{2} h^2 \cdot \lambda_{sy}$$

indicates the waiting times of additional passengers $h \cdot \lambda_{sy}$ that arrive at stop s between the time that trip n passed by stop s and trip $n + 1$ arrives at that stop. It is split into two sub-terms: $h \cdot \lambda_{sy}$ that indicates the number of additional passengers and $\frac{1}{2} h$ that indicates the average waiting time of an arriving passenger assuming uniformly distributed passenger arrivals. Note that this waiting time does not depend on our decision variable, x_s, which indicates whether trip n skips stop s or not. For this reason, even if we included it in the objective function for the sake of completeness, one can exclude it because it is a non-varying term.

The second component of the objective function,

$$M \sum_{s \in S} [u_s + (1 - x_s)]^2$$

penalizes progressively stops that are repeatedly skipped by successive trips. The progressive penalization is achieved by considering the square value of $u_s + (1 - x_s)$ in the objective function. For instance, if stop s was served by the previous trip, $n - 1$, then $u_s = 0$ and $[u_s + (1 - x_s)]^2$ will be equal to 1 if trip n skips stop s. If, however, stop s was served by trip $n - 2$, but not by trip $n - 1$, then $u_s = 1$ and the penalty to the objective function when trip n skips stop s will grow disproportionately from 1 to $[u_s + (1 - x_s)]^2 = 4$. This penalty term strives to alternate between skipped stops to not allow the same stop of the line to be skipped repeatedly by consecutive trips. For instance, if $[u_s + (1 - x_s)]$ was not squared, there would have been no difference between skipping a single stop 4 consecutive times or skipping 2 stops by 2 times each. To summarize, to improve the service coverage, we would like successive trips to skip different stops as much as possible.

Finally, we note that the penalization of the skipped stops

$$\sum_{s \in S} [u_s + (1 - x_s)]^2$$

is multiplied by a large positive number M which is a hyper-parameter of our mathematical model. This large number is used to provide higher

priority to the alteration among different skipped stops from trip to trip. Without this, the second component of the objective function would have had a minimal impact because the first component related to the passenger waiting times would have been the dominant one. In practice, to ensure that the second component of the objective function has higher priority, different values of M should be tested when solving the model.

In addition to our model's objective, we have a number of constraints. Constraint (2) ensures that the passenger load of trip n at any stop s is lower than the pandemic-imposed capacity limit, g. Constraint (3) returns the number of passengers that board vehicle n at the first stop of the trip, $s = 1$. Note that if trip n skips stop 1, then $\gamma_1 = x_1 \sum_{y \in S} p_{1y} = 0$. Constraint (4) returns the passenger load of trip n when it departs from stop $s \in S | 1 < s < |S|$. This is equal to

$$\gamma_s = \gamma_{s-1} + x_s \sum_{y \in S | y > s} p_{sy} - \sum_{y \in S | y < s} p_{ys} \cdot x_y$$

where:

- γ_{s-1} is the passenger load at the previous stop $s - 1$,
- $x_s \sum_{y \in S | y > s} p_{sy}$ are the passengers boarding trip n at stop s,
- and $\sum_{y \in S | y < s} p_{ys} x_y$ are the alighted passengers at stop s.

The alighted passengers are the sum of passengers who boarded at a previous stop $y \in S | 1 \leq y < s$ and alight at stop s, $p_{ys} \cdot x_y$, where x_y indicates whether vehicle n skipped stop y or not. Reckon that all on-board passengers with stop s as their final destination will be allowed to alight at stop s even if stop s is skipped.

Constraint (5) ensures that trip n will board passengers at, at least, one stop of the line $\langle 1, 2, \ldots, |S| - 1 \rangle$. This constraint makes sure that trip n will not be canceled.

3.2. Model properties and computational complexity

Our service pattern model expressed in Eqs. (1)–(6) considers the pandemic-imposed capacity limit in its constraints and it is an integer nonlinear programming problem (INLP) with a quadratic objective function and linear constraints. This model needs to be solved every time a new vehicle is about to be dispatched. Due to its combinatorial nature, the problem can be solved to *global optimality* with an exhaustive search of the solution space. When using exhaustive search, its computational complexity is $O(2^{|S|})$ according to the big O notation. Therefore, exploring the entire solution space with brute force results in exponential computational complexity. This is formally proved in Lemma 3.1.

Lemma 3.1. The dynamic service pattern problem subject to a pandemic-imposed capacity has an exponential computational complexity that requires exploring $2^{|S|}$ potential solutions.

Proof. The decision variables that determine the service pattern, x_s, can receive binary values. Formally, to find the globally optimal service pattern of a vehicle n, we need to explore a set of $2^{|S|}$ potential solutions because at each stop $s \in \{1, 2, \ldots, |S|\}$ we have two options: serve or skip ($x_s \in \{0, 1\}$). Ergo, the potential solutions that need to be evaluated are $2^{|S|}$.

As also demonstrated in Fu et al. (2003), solving this problem to global optimality with brute-force is possible in public transport lines with realistic sizes. If, however, the number of stops is unrealistically high then we cannot evaluate every potential service pattern with the use of brute-force and we need to resort to sub-optimal solution approximations with the use of heuristics. Fig. 1 provides an indication of the increase of the solution space with the number of stops indicating also the possible computations that can be executed in 1 min by the world's fastest supercomputer that can execute up to 33,860 trillion calculations per second. We note that we select the computation limit
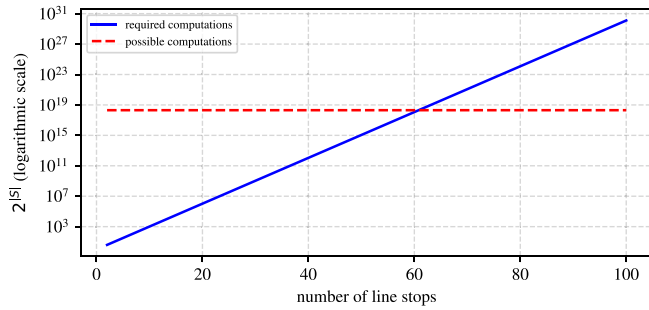
Fig. 1. Required solution evaluations when the size of the line, $|S|$, varies.

of one minute because the decision about the service patter of each vehicle should be made within a limited time during which the vehicle awaits to be dispatched. Under this condition, a solution can be computed for lines with up to 60 stops (see Fig. 1).

3.3. Demonstration

To demonstrate the application of our model, we consider a toy network with $S = \langle 1, 2, 3 \rangle$ stops. Stops 1 and 3 are not skipped by the previous trip, whereas stop 2 is skipped by the two latest trips of the service line. That is,

$$(u_1, u_2, u_3) = (0, 2, 0)$$

as it can be also seen in Fig. 2.

Our model will decide which stops should be skipped by the current trip n. In this scenario, the number of passengers waiting at each stop when trip n arrives is presented in Table 1.

Let also the average passenger arrival rate at stop s for passengers with destination y be equal to $\lambda_{sy} = 1$ passengers per 2 min for all $s \in \langle 1, 2 \rangle$ and for all $y \in S | y > s$. The planned time headway between successive trips of the line is considered to be $h = 5$ minutes. In addition, the value of M that penalizes the consecutive skipping of the same stop is set equal to 1.

We also consider the following cases to examine the solutions of our model:

- case I: the pandemic-imposed vehicle capacity is $g = 30$ passengers
- case II: the pandemic-imposed vehicle capacity is $g = 20$ passengers

In the first case, the pandemic-imposed vehicle capacity is enough to accommodate all passenger demand. As expected, in that case our model finds the following optimal solution:
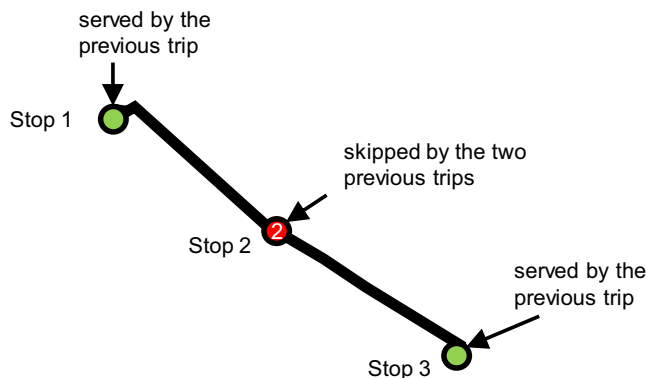

Fig. 2. Presentation of the stops of the line indicating which stops are served and which are skipped by the previous trips of trip n.

Table 1

p_{sy}: number of passengers waiting at stop s that are willing to travel to stop y at the time trip n arrives at stop s, where s is the origin and y the destination.

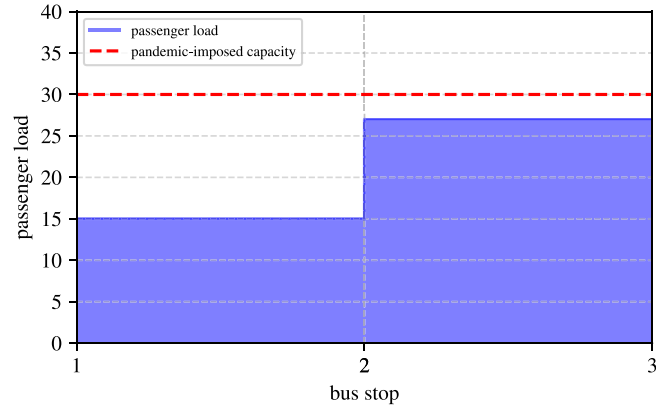| Origin	Destination		
	1	2	3
1	0	7	8
2	0	0	19
3	0	0	0

Fig. 3. Passenger load from stop 1 to stop 2 and from stop 2 to stop 3 in case I.

$$(x_1, x_2, x_3) = (1, 1, 1)$$

which indicates that all stops will be served. The resulting in-vehicle passenger load after the departure from each stop is presented in Fig. 3. One can note that this load is below the pandemic-imposed capacity limit of 30 passengers.

In addition, the aggregate passenger waiting times until the following vehicle $n + 1$ arrives at stop s are

$$\sum_{s \in S} \sum_{y \in S | y > s} \frac{1}{2} \left[(u_s + (1 - x_s)) \cdot h \cdot p_{sy} + h^2 \cdot \lambda_{sy} \right]$$

$$= 113.75 \text{passenger} - \text{minutes}$$

In case II, where the pandemic-imposed capacity limit is not enough to serve all passenger demand, the model returns the following optimal solution:

$$(x_1, x_2, x_3) = (0, 1, 1)$$

indicating that the first stop of the line will be skipped. This is also an expected outcome since stop 2 was skipped by the two previous trips and receives higher priority when making a decision about which stops to serve in case we cannot serve all of them. The resulting in-vehicle passenger load after the departure from each stop is presented in Fig. 4 and one can note that this passenger load is below the pandemic-imposed capacity limit of 20 passengers.

In addition, we have $7 + 8 = 15$ unserved passengers that were skipped at stop 1. The aggregate passenger waiting times until the following vehicle $n + 1$ arrives at stop s are:

$$\sum_{s \in S} \sum_{y \in S | y > s} \frac{1}{2} \left[(u_s + (1 - x_s)) \cdot h \cdot p_{sy} + h^2 \cdot \lambda_{sy} \right]$$

$$= 151.25 \text{passenger} - \text{minutes}$$

4. Numerical experiments

4.1. Case study description

Our case study is bus line 9 in the Twente region operated by Keolis Nederland. The bus line connects two cities: Hengelo with 80 thousand
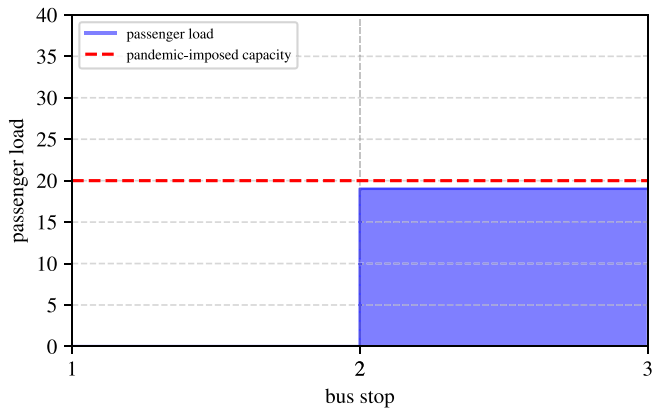
Fig. 4. Passenger load from stop 1 to stop 2 and from stop 2 to stop 3 in case II.

inhabitants and Enschede with 160 thousand inhabitants. This line is selected because it also serves the University of Twente (UT) which is approximately in the middle of the two cities. The line consists of 13 stops per direction and the stops that accommodate the university (Enschede Kennispark/ UT and Enschede Westerbegraafplaats/ UT). The topology of the line is presented in Fig. 5.

The average trip travel time per direction is 16 min. In addition, the operational hours of the line service are presented in Table 2 and the value of M that penalizes the consecutive skipping of the same stop is set equal to 10000 because this ensures the prioritization of the second term of the objective function.

4.2. Model application

In our case study we focus on the peak time period of weekdays, which is from 8 am to 9 am. The mean passenger demand in this period is presented in the origin–destination demand matrix of Table 3. As in Welding (1957), Randall et al. (2007), it is hypothesized that the passenger arrivals at stops are random, following a uniform distribution because of the high service frequency (see Assumption 1 of our mathematical model). During this time period, the planned headway among successive trips is $h = 5$ minutes. The pandemic-imposed
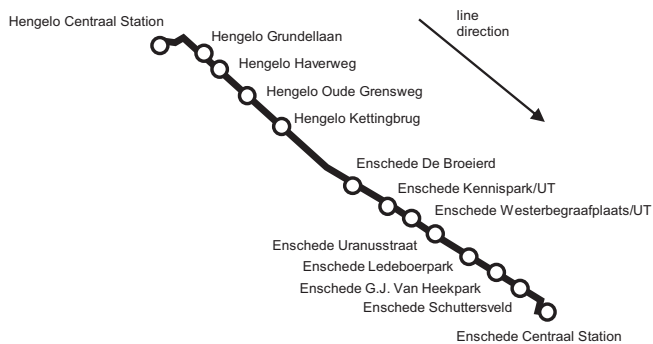

Fig. 5. Topology of bus line 9 in Twente, Netherlands.

Table 2
Operational hours of bus line 9 (line direction: Hengelo centraal → Enschede centraal).

Day	Operational hours
Weekday	06:29–23:29
Saturday	07:29–23:29
Sunday	10:29–23:29

capacity is $g = 59$ passengers (38 seated, 21 standees), and the nominal (pre-pandemic) vehicle capacity was 81 passengers (38 seated, 43 standees).

In addition, we select a particular trip n which is about to be dispatched and for which we should determine a service pattern. Before trip n, we had the following consecutive bus stop skips presented in Fig. 6:

In our experiments, we compare the performance of the following models:

(a) the *as-is* case where trip n serves all stops without applying a service pattern;
(b) the *SPS - nominal capacity* service pattern model that considers the nominal (pre-pandemic) capacity instead of the pandemic-imposed capacity when solving the model in Eqs. (1)–(6);
(c) the proposed *SPS - pandemic capacity* service pattern model that considers the pandemic-imposed vehicle capacity expressed in Eqs. (1)–(6).

Our *SPS - pandemic capacity* is programmed in Python 3.7 using a general-purpose computer with Intel Core i7-7700HQ CPU @ 2.80 GHz and 16 GB RAM. The software code of our model is publicly released at TU.ResearchData (2020) and the mathematical model is solved with Gurobi 9.0.3. The *SPS - nominal capacity* is also programmed in Python 3.7 and its main difference is that it replaces the pandemic capacity constraint:

$$\gamma_s \leqslant g, \ \forall s \in S$$

by constraint:

$$\gamma_s \leqslant \tilde{g}, \ \forall s \in S$$

where $\tilde{g}$ is the nominal capacity of 81 passengers. The optimal service pattern solutions of the *SPS - pandemic capacity* and the *SPS - nominal capacity* models are computed with our publicly released software code. The optimal service pattern solutions are presented in Fig. 7. Note that *SPS - pandemic capacity* results in an increased number of skipped stops. Note also that these skipped stops are close to the first stop because they were served by the previous trip $n - 1$ and had a higher probability to be skipped by trip n.

4.3. Evaluation

The optimal service patterns presented in Fig. 7 are computed deterministically assuming that the realized passenger demand will be equal to the expected demand values presented in Table 3. In reality, however, the realized passenger demand might vary from our expectations even if our decisions refer to the short future. For this, we generate 1000 origin–destination demand scenarios by sampling demand values from a *restricted* normal distribution that restricts the sampling of negative demand values. This normal distribution uses as mean the presented hourly demand values in Table 3, and as standard deviation the respective mean value multiplied by 30% since our short-term demand predictions cannot deviate significantly from the realized demand.

To evaluate the performance of the aforementioned methods, we use the following key performance indicators:

(i) $\mathscr{O}_1$: the total passenger load of vehicle n that exceeds the pandemic-imposed capacity at all stops;
(ii) $\mathscr{O}_2$: the number of unserved passengers by vehicle n that have to wait for the next vehicle, $n + 1$;
(iii) $\mathscr{O}_3$: the total waiting time of unserved passengers by trip n that will have to board the next vehicle, $n + 1$.

The aforementioned key performance indicators will assess the benefits of our proposed *SPS - pandemic capacity* model, while evaluat-

Table 3
Mean values of the hourly origin-destination demand matrix from 8:00 until 9:00.

Origin stop	Destination stop												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1		8	16	16	24	24	16	16	24	16	8	32	44
2	×	×	4	8	8	16	16	8	16	24	40	24	52
3	×	×	×	4	4	4	32	24	16	16	32	40	32
4	×	×	×	×	8	8	16	24	32	40	8	24	56
5	×	×	×	×	×	4	8	8	20	20	8	28	28
6	×	×	×	×	×	×	8	4	8	24	12	20	32
7	×	×	×	×	×	×	×	4	4	4	12	24	48
8	×	×	×	×	×	×	×	×	8	8	4	8	36
9	×	×	×	×	×	×	×	×	×	4	8	16	44
10	×	×	×	×	×	×	×	×	×	×	4	12	48
11	×	×	×	×	×	×	×	×	×	×	×	8	12
12	0	×	×	×	×	×	×	×	×	×	×	×	4
13	×	×	×	×	×	×	×	×	×	×	×	×	×
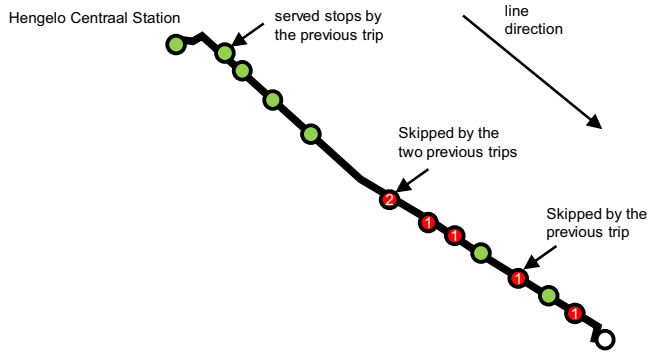
Fig. 6. Presentation of the stops served by the previous trip, $n-1$ (green color), the stops skipped by trip $n-1$ (red color), and the stop skipped by two consecutive trips (red color).

Fig. 8. Tukey boxplot of the performances of the *as-is*, *SPS - nominal capacity*, and *SPS - pandemic capacity* designs in 1000 scenarios.

ing also its potential externalities, such as the increase in unserved passengers and the increase in passenger waiting times at stops for passengers that are skipped by vehicle *n*.

First, the results of our implementation with respect to the total in-vehicle passenger load that exceeds the pandemic-imposed capacity, $\mathscr{O}_1$, are presented in Fig. 8. Note that this key performance metric sums the number of passengers that exceed the pandemic-imposed capacity when vehicle *n* departs from every stop. In Fig. 8 we use the Tukey boxplot convention (see McGill et al. (1978)) to present the results from the 1000 scenarios. In the Tukey boxplot, the upper and lower boundaries of the boxes indicate the upper and lower quartiles (i.e. 75th and 25th percentiles denoted as Q_3 and Q_1, respectively). The lines vertical to the boxes (whiskers) show the maximum and minimum values that are not outliers. The whiskers are determined by plot-
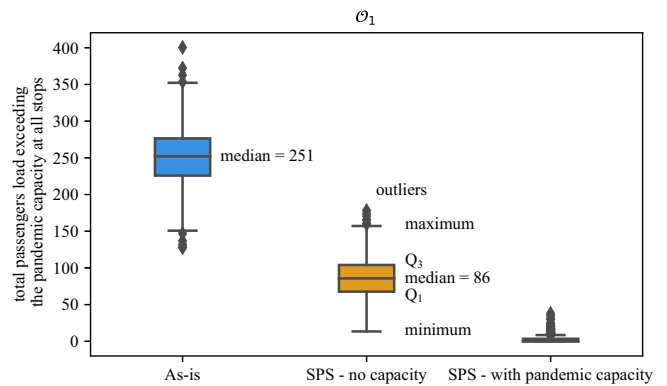
ting the lowest datum still within 1.5 of the interquartile range (IQR) Q_3-Q_1 of the lower quartile, and the highest datum still within 1.5 IQR of the upper quartile.

From Fig. 8 one can note that the *SPS - pandemic capacity* service pattern results in passenger loads that do not exceed the pandemic-imposed capacity (the median value is almost equal to 0). This is an expected outcome since our service pattern model incorporates the pandemic-imposed capacity as a hard constraint and this capacity can be only exceeded if the realized demand differs significantly from the expected one. The worst performance is observed at the *as-is* design which does not skip any stops resulting in a median passenger load of 251 passengers beyond the pandemic-imposed capacity. One interesting observation is that the *SPS - nominal capacity* service pattern, which only skips stops 2 and 4, improves this key performance
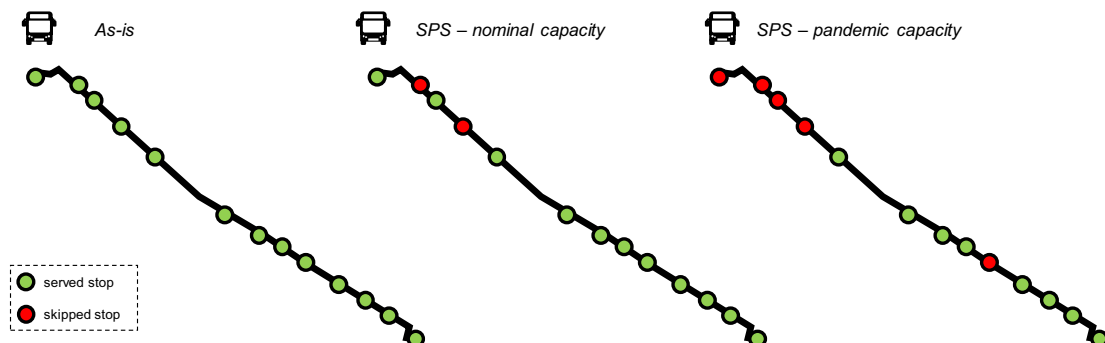

Fig. 7. Optimal service patterns of trip *n* based on each method.

indicator by almost 65% compared to the as-is service pattern. It still results though in considerable crowding beyond the pandemic-imposed capacity limit.

Besides this aggregate analysis, it is also important to investigate which stops are the problematic ones that lead to excessive passenger loads. For this, in Fig. 9 we also report the average values of the passenger loads from the 1000 scenarios after the departure of vehicle n from each stop $s \in \langle 1, 2, \ldots 13 \rangle$.

The results of Fig. 9 justify the *SPS - pandemic capacity* solution that skips stops 1, 2, 3, 4 and 9. By skipping stop 9, the passenger load remains below the pandemic-imposed capacity limit of 59 passengers at stops 9–13. In addition, skipping stops 1–4 results in a reduced passenger load between stops 5–9. The *SPS - nominal capacity* violates the pandemic-imposed capacity constraint already in stop 6 and this continues until stop 12. Even worse, if vehicle n allows passengers to board at any stop by adopting the *as-is* design, we would exceed the pandemic-imposed capacity from the 4th stop.

It is important to note that the as-is case results in passenger loads of more than 59 passengers between stops 4–12, thus increasing considerably the risk of virus transmission. In addition, this design exceeds the pandemic-imposed capacity limit by more than 50 passengers when traveling from stop 8 to 9. The *SPS - nominal capacity* service pattern has improved performance compared to the as-is design and exceeds the pandemic-imposed capacity limit when traveling from stop 6 until stop 12. It also results in 20 more passengers than the pandemic-imposed capacity limit at stop 8. However, as expected, its passenger load remains below the nominal capacity limit of 81 passengers.

In terms of exceeding the pandemic-imposed capacity, it is clear that our proposed *SPS - pandemic capacity* solution performs significantly better than the alternative designs. This, however, comes at a cost because the *SPS - pandemic capacity* service pattern skips five stops resulting in increased passenger waiting times and unserved demand. The number of unserved passengers should be examined because we refuse service to passengers who want to board at the skipped stops 1–4 and 9. To compare the numbers of unserved passengers by vehicle n, we implement the three service patterns in the same 1000 scenarios with different origin–destination demand and we report the aggregated results in the boxplots of Fig. 10.

One can note that the *SPS - pandemic capacity* service pattern results in a median of 69 unserved passengers by vehicle n. This is a very useful result for public transport operators because it will help them quantify the cost of unserved demand when trying to maintain the pandemic-imposed vehicle capacity. It is also of interest to understand which are the most problematic stops where passengers are refused boarding. For this, we also report the average number of unserved passengers at each stop in Fig. 11.
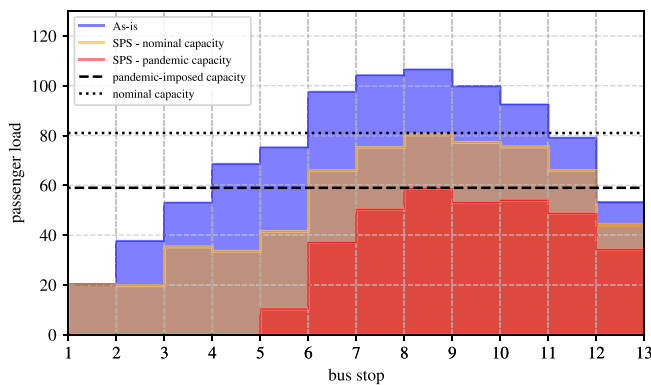

Fig. 9. Average passenger load when departing from each stop when implementing the service pattern with nominal capacity, with pandemic capacity, and the as-is design.
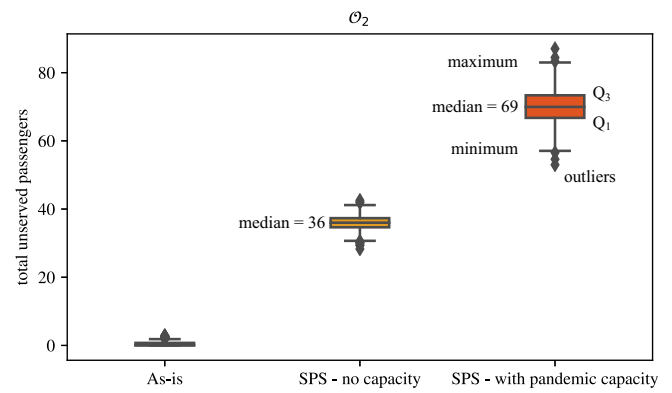

Fig. 10. Tukey boxplot of the unserved demand by vehicle n for each service pattern implemented in 1000 scenarios with random passenger demand.
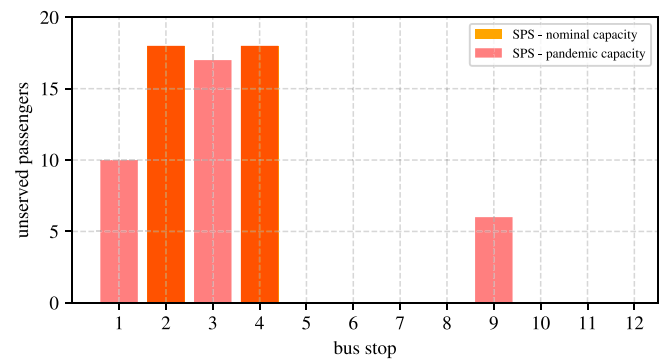

Fig. 11. Average number of unserved passengers by vehicle n at each stop when implementing the service pattern with nominal capacity and with pandemic capacity.
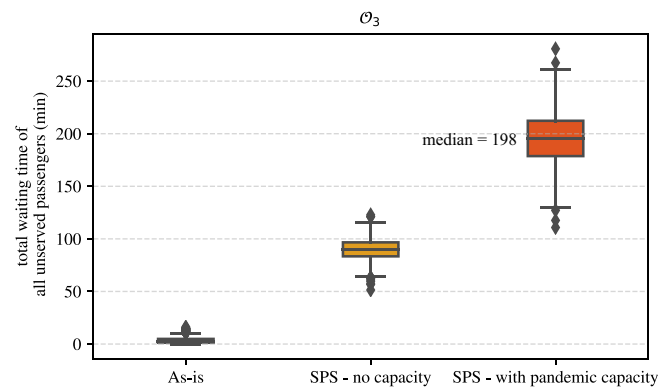

Fig. 12. Tukey boxplot of the extra passenger waiting times of unserved passengers by vehicle n.

From Fig. 11 one can note that the unserved passengers of the service pattern solution that considers the nominal capacity and the solution that considers the pandemic-imposed capacity is the same for stops 2 and 4. The reason for this is that both solutions skip stops 2 and 4. When implementing the *SPS - pandemic capacity* solution, we have additional unserved passengers at stops 1, 2, and 9 because these stops are skipped to maintain a passenger load below the pandemic-imposed capacity limit. We finally note that Fig. 11 does not include the results from the *as-is* design, because this design does not skip any stops.

The increased passenger waiting times expressed in the key performance indicator $\mathcal{O}_3$ are mostly analogous to the results of the unserved demand because the more unserved passengers, the higher their total waiting time. The results with respect to the key performance indicator $\mathcal{O}_3$ are presented in Fig. 12 and, as expected, are mostly in-line with the results of Fig. 10. The median of the total waiting time of unserved passengers is 198 min when implementing the service pattern *SPS - pandemic capacity*.

5. Concluding remarks

5.1. Discussion

In this work, we developed a service pattern model for determining dynamically the skipped stops of a public transport vehicle that is about to start its service in order to satisfy the pandemic-imposed capacity. Several public transport authorities in major cities have already used service patterns to avoid overcrowding but those decisions are made offline and are not vehicle-specific (e.g., they result in daily stop closures). Our service pattern model filled this research gap and can be applied in dynamic environments by using up-to-date estimations of passenger demand. One of its benefits is that it can be applied in near real-time for public transport lines with realistic sizes to return an optimal solution.

Although service patterns are used by many public transport operators, one should consider that by skipping particular stops the number of unserved passengers and their waiting times might increase. To evaluate these negative effects, we implemented our model in a bus line connecting two cities with the University of Twente. We also implemented the as-is service that does not skip any stops and a service pattern model that does not try to meet the pandemic-imposed capacity limit. The results of our evaluation demonstrated that our model's solution can reduce the total passenger load that exceeds the pandemic capacity by 251 passengers per vehicle (or 20 passengers per vehicle per stop). Importantly, this service pattern is able to maintain a passenger load below the pandemic capacity limit in almost all cases. This, however, results in skipping a considerable number of stops (5 out of 13 stops) and a significant number of unserved passengers that can be up to 69 passengers per trip.

The aforementioned values are based on the setting of our case study. Our service pattern model that considers the pandemic-imposed capacity might perform significantly better in public transport lines with relatively low passenger demand because potential skipped stops will not result in many unserved passengers. Additionally, lines with small headways are more suitable for the implementation of service patterns since travelers will not have to wait for an extended period of time if they are skipped by a vehicle. To summarize, our decision support model can propose service patterns for different line services and can help public transport operators to assess the benefits and drawbacks of implementing pandemic-driven service patterns given their operational headways and their passenger demand levels.

5.2. Future directions

In future research, our service pattern model can be expanded to consider more preferences from the operational side of a service line, such as the reduced vehicle travel times due to the skipped stops. One important research topic is also the combination of our model with dynamic frequency models that can increase the number of vehicles at particular time periods of the day to offer an increased vehicle supply to high-demand stops skipped by our service patterns. In case of limited vehicle availability from the public transport operator, on-demand services and shared mobility options can complement this supply gap by combining our model with models for on-demand scheduling or vehicle sharing.

5.3. Limitations

One of the limitations of our work is that stop service patterns result in passengers' inconvenience because of the skipped stops. Given that our model tries to avoid skipping the same stop repeatedly, in high-frequency services this might not be a major problem. However, our approach might be not practical for low-frequency services, such as long-distance trains. In addition, skipping stops results in higher concentrations of waiting passengers at the skipped stops. If the stops are outdoors, this is not a major problem in terms of virus transmission. If, however, we refer to stops in indoor spaces (e.g., metro stations), our approach might not result in a virus transmission improvement.

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