Future Generation Computer Systems 100 (2019) 1088–1102

Contents lists available at ScienceDirect

Future Generation Computer Systems

journal homepage: www.elsevier.com/locate/fgcs

Stackelberg model based game theory approach for assortment and selling price planning for small scale online retailers

Zahra Saberi [a,*], Morteza Saberi [b], Omar Hussain [a], Elizabeth Chang [a]

[a] School of Business, UNSW Canberra, Australia
[b] School of information systems and modeling, University of Technology Sydney, Australia

HIGHLIGHTS

- A novel assortment planning model for small scale online retailers is proposed.
- Develops strategies to deal with barriers arising from large-scale retailers.
- An effect of the consumer's basket shopping is considered in the developed AP model.
- Determines the assortment type, selling price and items quantity to be ordered.
- Considering shopping basket changes the pricing and AP model's recommendation.

ARTICLE INFO

Article history:
Received 9 February 2019
Received in revised form 19 April 2019
Accepted 26 May 2019
Available online 29 May 2019

Keywords:
Assortment planning
Small scale online retailers
Supplier uncertainty effects
Stackelberg game-theoretic approach

ABSTRACT

Assortment Planning (AP) is one of the most significant and challenging decision for online retailers (e-tailers) to make. This decision becomes even more complex when a supplier is considered as a distinctive participant in decision making model. In the bricks and mortar mode of retailing, retailers are more powerful than suppliers in getting the required goods in the required quantity. However, this is not the case for small scale e-tailers. Such e-tailers are faced with situations where large-scale retailers indirectly force the suppliers to refuse supplying to them. In such cases, effective AP decision making approaches are needed for small scale e-tailers to get the required goods to satisfy the customers' demand. While current advancement in smart cities provide a powerful platform and support for successful operations of online retailing, this needs to be supported by appropriated modeling approaches that assist the e-tailer in getting their required product assortment. In this paper, a game-theoretic model is developed to support the small scale e-tailer in AP decision making. Such that it has two strategies to decide from. The first strategy is that it can offer the product with supreme quality by procuring it from the main powerful supplier and the second strategy is to offer the product from a less popular brand. The first strategy is modeled as a non-cooperative Stackelberg supply chain in which the supplier plays a leader and the e-tailer is a follower and the second strategy is modeled as an assortment planning problem while considering utility degradation of providing alternative brand to the customers. Various analyses are done to find the best strategy in different scenarios before recommending the best strategy to be followed by the e-tailer in given situations.

1. Introduction

For retailers, Assortment Planning (AP) is one of the fundamental decisions they have to make. Hence, AP is one of the key factors required by retailers to create a good image in front of customers besides location and price [1]. A retailer decides what and in what quantities to order products by which the customer demand, over a period, can be satisfied. During assortment planning, retailers need to optimize their assortment variety and size by balancing between offering a limited and wide product assortment. Carrying a limited assortment may keep the operational costs low but cannot meet the expectations of customers seeking variety. On the other hand, carrying a wide assortment not only causes variety fatigue effect but also increases the capital and operational costs as well as increasing the risk of product obsolescence. So, the challenge for the retailers during AP is to pre-determine customers' preferences along with other constraints like space and budget in making decisions that will have great economic impact on their business [2].

One of the newest types of retailing is electronic retailing also known as e-tailing. E-tailing which is important for smart cities, can be classified into the categories of either large scale or small

* Corresponding author.
E-mail address: z.saberi@student.adfa.edu.au (Z. Saberi).

https://doi.org/10.1016/j.future.2019.05.066
0167-739X/

scale retailers. Large scale e-tailers such as Amazon have a wide range of resources at their disposal thereby allowing them to offer a wide variety of goods. The small scale e-tailers do not have these and, hence, need to make smart decisions in planning their assortment in the constraints available to them. Although there is a growing trend for online shopping [3], there are only a few studies that focus on e-tailing's assortment planning. It is important to note that due to their structure and retailing strategy, AP in e-tailing is quite different than in traditional retailing with bricks and mortar stores. As the online mode of retailing has increased the customers' awareness and expectations of their shopping experience, e-tailers need a wider assortment in comparison to bricks and mortar retailers to enhance the customers' shopping experience. Also, traditional retailing is more attractive and enjoyable for most customers compared to online shopping [4], e-tailers should provide a more dynamic and customized product assortment to attract customers.

One of the main challenges of e-tailing is deciding the assortment type, especially considering supplier uncertainty. Supplier uncertainty refers to the possibility of the online retailer not getting the required goods from the preferred supplier. Although there has been a power transformation in retailing which means that retailers generally have more power than suppliers [5], this is not corrected in small scale online retailers. With the realization of smart cities, citizens have access to fast internet, good transportation and smart phones for achieving their objectives. However to fully realize such benefits, one of the barriers specifically for online small scale retailers, is the need to have effective strategies by which they can deal with well-established bricks and mortar retailers on the market. It has been mentioned in the literature that for such online retailers, strong bricks and mortar brands force the supplier indirectly to refuse supplying to them [6]. In fact, small scale online retailers had difficulty to prove their reliability to powerful suppliers. As mentioned by Kumar [7], the balance of power between manufacturers and retailers has shifted with the retailers having the upper hand. With this, some retailers force the supplier directly or indirectly to refuse supply to online retailers that may be their competitor as shown in Table 1. This complicated the AP for online retailers where, on the one hand, there is a need for more variety and customization of their assortment while on the other hand they do not have not enough power in interacting with suppliers to get the desired assortment of items.

Another significant aspect of AP is considering basket shopping by consumers. Basket shopping takes into consideration the different products that customers buy from the e-tailer. It is clear that online customers prefer to purchase many items in a transaction to avoid being charged multiple delivery fees by the e-tailer [8]. As Cachon and Kök [9], Ghoniem, et al. [10] and Mani, et al. [11] discussed, the tendency of customers to purchase a basket of items should be considered in AP to prevent getting sub-optimal results. Usually, basket shopping consumers consider the overall basket utility in purchasing. The weight of importance that customers assign to items in the shopping basket could vary in different customers. Mani, et al. [11] consider the items' price as the important weight. Chen, et al. [12] consider an extreme case that just one product is important for customers in purchasing from the store. The authors labeled this product as the "lead category" that attracts customers for shopping from the particular e-tailer. This means customers consider the utility of the lead product in shopping.

However, from the perspective of the small scale e-tailer combined with supplier uncertainty, if the product in question is the lead product, the small scale retailers will have to decide: (a) the impact on its sales if it substituted the required product with an alternative product type from a different supplier; (b) to determine the assortment and quantity in which it needs to have that product; and (c) the price at which it should sell that product to the customers.

A variant of this problem is when the e-tailer decides to substitute a lead product's brand with another one that has more easily accessible suppliers. In other words, it is procured from perfect completion market. We model these utility reductions and help an e-tailer make the best decision in different scenarios. Although there is a growing literature on AP, these uncertainties have not been discussed and addressed in current literature.

In considering the affect of supply chain participants in e-tailer operation decisions, the conflict of interests and power of various agents should be considered. Unlike most supply chain decision making practices that explore the problem from a centralized view, we consider the new paradigm more practical. This type of interaction calls for leader–follower game theory based modeling. We use the Stackelberg game to solve the problem. To the best of our knowledge this is the first work that expands previous models to include bringing supplier effect on assortment planning decision making by using game theory as a solution.

The rest of the paper is organized as follows. The relevant literature is presented in Section 2. The problem definition is explained in Section 3. Section 4 discusses problem notations and formulations. Game theory-based solution methodology is presented in Section 5. Numerical examples to demonstrate the application of game theory based approach in AP is shown in Section 6. Finally, the conclusion highlights the innovation of the model and conclude with the future work.

2. Background and literature review

The Assortment Planning problem is one of the important and challenging decisions in retailing. As mentioned in Section 1, AP determines the number of different category of products to carry (the width) for sales, the different product lines to carry in each category (the breadth), as well as details and inventory levels for each product line [13]. The goal of AP is to determine the set of product assortments to have in stock while considering retailer constraints like shelf space or available budget.

AP literature has affinity with two other fields of research in the literature, namely, product line design in the marketing area and multi-item inventory problems [14]. However, it has its differences too. We briefly discuss the similarities and differences of AP fields with the two mentioned research fields. In product line design, the manufacturer determines the attribute values for the portfolio of production items that are vertically differentiated like different features and quality levels [15], whereas in AP, retailers determine the assortment of products from different suppliers which are horizontally differentiated which means they are similar in quality but they are different in size and color. In multi-item inventory planning problems, the inventory levels of multiple products is determined according to the budget or space limitation without having product selection whereas, in AP, product selection and their level of inventory are among the decisions variables [14].

As our focus in this paper is on AP, we only focus on literature in this area. Demand estimation modeling is the backbone of the AP problem. Customers value product variety and assortment from the retailer and, hence, they exert a positive affect on customer demand. However, capturing the demand of product assortment is not easy as various parameters such as the size of the assortment, taste of customers, availability of a substituted assortment and so forth, have an affect on that. One of the main factors that have emphasized its role in the literature is substituted assortment. Substitution can either be an assortment based or stock-out based. Assortment (static) based substitution occurs

Table 1
Small scale online retailers' problems in facing strong suppliers: Industrial experiences [6].

Author	Role	Statements
Rod Sims	Australian Competition and Consumer Commission Chairman	"We're looking at online businesses that are being held back through a variety of means, not just price maintenance but also refusal to supply."
Wai Hong Fong	Chief Executive of online retailing group: OzHut	"They will ask if you sell online, and then if you do, they will say, 'we don't sell to online, we can't open an account.'" "When we started, most of the sellers didn't want to give us anything. We qualified for both categories – online and start-up – so it was incredibly difficult."
Sue Cook	Director of Multichannel at Taos Creative	"There is still a large portion of customers who research but don't actually buy online," says Cook. "Therefore, even supplying to online retailers will not entail the demise of bricks-and-mortar channels."

when the retailer does not have the first choice of customers. Stock-out based substitution occurs when although retailers carry the first choice of the customer, at the time of customer purchasing, that assortment is finished. Demand estimation models in AP literature are generally performed by using the following models: Multinomial logit (MNL) [16], Locational choice [17] or Exogenous demand [18]. Initial work on AP in the literature has evolved to improve the accuracy in estimating customer choice. We refer to the Kök, et al. [14] for a good explanation source.

One of the initial work on AP is by Ryzin and Mahajan [16] in which they present the AP problem of the intersection of operations research and marketing. They model the uncertain choice behavior of customers in a more detailed level using MNL models. This is the point of which AP sets its own way of modeling the demand as opposed to how it was done in the field of product line selection model demand deterministically in an aggregated level [14]. In modeling the problem, the authors have considered price as an exogenous variable and inventory and assortment size as variable decisions. The authors state that optimal assortment is always the most popular one that has the highest utility from a customer perspective. This initial model was improved in further work [19] by considering the dynamic substitution of customer behavior. By the term of dynamic substitution, we mean that customer choice is also dependent on the inventory level of the item in the store. Smith and Agrawal [18] model the product assortment and inventory planning using an **exogenous demand** model rather than using MNL. In an **exogenous** model, the demand rate is directly specified. Along with substitution probability rate between items in the case of stock-out indicated in the matrix form. The authors show the effect of considering substitution in the planning of inventory and selecting items concurrently on the profit. Also, they show that the optimal assortment is not among the most popular one when product demand rates are different. Cachon, et al. [20] extend Ryzin and Mahajan [16] study by considering consumer search in AP planning. Consumer search arises from the fact that they are uncertain about the decisions, taste or the stores in which the shopping should be done. So, they search other stores in the hope of finding a better product. The authors consider two opposing assumptions in their work; independent search and dependent search. Independent search refers to the cases where the assortment of each store is unique such as a jewelry store. Dependent search refers to the cases where the assortment in stores has overlaps. They concluded that the final assortment includes ones that are not very profitable products. This finding is in contrast with the Ryzin and Mahajan [16] results and findings.

Gaur and Honhon [17] for the first time proposed a **locational demand model** based on the Lancaster and Hotelling models [21,22]. Locational demand model considers products as a bundle of several attributes and locates them based on the value of attributes. In a locational model, customers' preferences are considered on product features rather than itself as opposed to the MNL model. The main concept behind the locational demand model is that where a product attribute space is constructed, a point, which represents customers' preference, based on the used attribute, is pinned in the space for each customer. The customer behavior is simulated in a way that customers just select the closest product located to them in the attribute space. The same reasoning causes the notion that distance is applied to determine the substitution product. They consider substitution of two types, namely static and dynamic. In the static substitution, the optimal assortment is located far from the other to cover all the attribute space. In dynamic substitution, it is near to the other as it is profitable to carry a wider assortment to allow substitution. Davis, et al. [23] developed nested logit model for demand modeling for AP. Nested logit model provides modeling correlation between choices in the set that is used in modeling customer brand loyalty. The authors show that the problem generally is NP-hard. However, it can be solved in polynomial time in some special cases.

During the time, researchers consider more variables in their models such as selling price, supplier selection and shelf space allocation are considered in AP models. In their model, Maddah and Bish [24] relax the assumption of price as an exogenous variable. They develop a model based on the MNL model and newsvendor inventory policy to optimize price and assortment decisions. They find the optimal structures of optimal assortment and propose an efficient solving heuristic producer.

AP literature is extended by considering other supply chain issues such as the number of views of a product by a customer, supplier selection, space planning and so forth. Kök and Fisher [25] developed a model to consider shelf space limitation using exogenous demand. They develop an iterative heuristic to find the optimal number of facings for each product and a local search to derive the space required for a subcategory. Hübner, et al. [26] improved on Kök and Fisher [25] model to a more time-efficient heuristic algorithm which out-performs in terms of optimality and computational time efficiency. Hübner [27] developed a decision support system for concurrent assortment and shelf space planning. In other work [28], authors developed a model to determine assortment and shelf space planning integrally. They considered that demand is space elastic, uncertain and proposed an efficient heuristic for large scale problems. Yücel, et al. [29] considered some operational issues such as shelf space constraint and supplier selection on modeling AP problems. The authors investigated the effect of each issue on the AP separately and showed that the AP result is inefficient by not considering these operational issues. Caro and Gallien [30] developed a model to consider uncertainty in customer preferences in AP. They proposed a model by which customer demand is learnt over a period and then by using this information, the demand over the remaining time is predicted. Their model is very suited for AP in fashion retailing because most of the items are new and have a short shelf life. It is evident that such stores need an agile supply chain to be able to implement the updated result of AP by procuring the new items. Sauré and Zeevi [31] model the dynamic AP problem by considering capacity constraint. The authors extended the classical multi-armed bandit algorithm to be applicable for assortment planning problems where the items' demand is dependent on the product set offered to the customer. Another research direction considers marketing issues and current perspectives such as customized assortment, competitive assortment or localized assortment. Customized assortment is the real-time planning and displaying of product assortments based on customer preferences and existing stock. This was developed by Bernstein, et al. [32] as well as Golrezaei, et al. [33]. Besbes and Sauré [34] investigated the modeling of AP in different market structures. Most of the analytics in AP have been carried out under the assumption that the market is a monopolist market. If customers searching by price as negligible, or customers have access to the information, considering a market as monopolist, then it is not real. They optimize assortment and pricing optimization in a duopoly competitive market. They developed an AP problem based on a multinomial Logit demand model and display space limitation. The authors showed that retailers carry a wider assortment in a competitive market. The importance of considering a shopping basket consumer and cross-selling affect is studied by Cachon and Kök [9], Ghoniem, et al. [10] and Mani, et al. [11]. Agrawal and Smith [35] extended their previous study to consider cross-selling between categories to optimize joint assortment and inventory. They assumed that customers prefer to buy a set of items and doing one of the following three actions in the case of stock-out: (a) buy a smaller basket, (b) substitute with a different basket, or (c) do not purchase at all. They showed that ignoring factors such as substitution behavior and customer basket preferences led to a significant decline in profit. Cachon and Kök [9] investigated the AP problem in a duopoly market structure for the two categories that are shopped together or separately by the customer. Customer choice is modeled with a nested logit model in which the customer first chose the store based on the assortment and then makes their shopping decision from both categories. AP in each category can be done in a decentralized manner or it can be done centrally for the entire category while centralized managing can lead to higher profit in the present of consumer basket, decentralize managing of each category lead to a higher price, lower variety and lower profit. However, centralize managing is so complex in reality because of a high dimension complexity in computation. The authors suggested using basket profit in front of accounting profit in decentralized managing of assortment to get near to the optimal centralize AP. Ghoniem, et al. [10] developed a model for joint assortment and pricing problems in the presence of cross-selling as a result of the complementary natures between items. They considered the asymmetric relation between complementary items that one of them is named as the primary category. The primary category is like coffee with sugar or cream as complementary categories. They modeled consumer choice using deterministic maximum-surplus model and proposed a mixed-integer nonlinear program that is solvable. Table 2 presents a summary and the literature on assortment planning. It can be seen that the focus of literature has been in different aspect that impact AP. None of them have taken into consideration the importance of supplier uncertainty of the effect it has on e-tailers. In this paper is a proposed approach to do with utilizing game theory as our modeling approach.

Game theory optimize (solve) the supply chain problems considering the interaction of different agents in supply chain structures. The interaction can be broadly categorizing uncooperative or cooperative decision making. For more information about game theory in supply chain we refer to Cachon and Netessine [36]. Uncooperative game, which means that each agent just considers their own profits, is countered with Stackelberg game which use in this paper, is a kind of uncooperative game. Stackelberg game application in supply chain problems is two broad categories: inventory planning, wholesale and retail pricing as well as promotion and marketing issues like cooperative advertising or national brand advertising [37]. We refer to Esmaeili, et al. [38] and Jørgensen and Zaccour [39] for more information. We define the problem we address in the next section.

3. Problem definition

Our problem is from the perspective of an e-tailer that has to decide about assortment planning by considering two types of uncertain actions. The first option is dealing with a supplier with no guarantee of it providing goods of the required assortment. The good being considered in our model is the lead product of the customer basket. In other words, this is the product that attracts customers to shop at the e-tailer. In such a case, the e-tailer has to make appropriate decisions of what to do in order to have either that item or a replacement item in stock. In answering that question, the second option is to determine what impact that will have on the e-tailer's operations if it substitutes the lead product with an alternate brand that is lower in quality. This is an alternate option that the e-tailer can choose.

In addressing such uncertainties, the e-tailer faces two options. The first is deciding whether to interact with the powerful supplier to carry products of a particular brand. The second option is a product of another brand with a lower level of quality. It may very well be the case that the alternate product may be higher in quality than the main product. However, in our work we consider that it is lower in quality and hence the e-tailer needs to decide according of what it needs to do in such scenarios.

The first strategy represents the non-cooperative Stackelberg game, which is composed of a supplier as a leader and the e-tailer as the follower. We define moves of the leader (supplier) as an intention to offer the lead product on an assortment unit cost and variety cost whereas selling price, purchase quantity, delivery service and assortment size of the lead item as the moves of the follower (e-tailer). The supplier announces its decisions about cost structures to the e-tailer. Cost structures are composed of two components: fixed cost (C) for adding variety and variable cost (c). The e-tailer makes its computation after observing these cost structures. In this game, the supplier's objective is to maximize its payoff functions while considering the follower optimal reaction to its moves [40]. Fig. 1 shows problem structures, related variables and decision process.

In the second strategy, the e-tailer should make an optimum decision in procuring the product of the alternate brand. We assume that the e-tailer has ease of access to this brand or quality level of product. So this is a typical AP problem in which the effect of quality reduction in the customer mind in substituting the main brand with an alternate brand and basket shopping behavior is considered.

In both strategies, an e-tailer jointly makes assortment size, price, and service level and quantity decisions in both scenarios. Finally, the e-tailer compares its final profit in each scenario to decide about supplier selection and assortment related decisions.

We also assume the specific inventory discipline in both scenarios, which is known as the Newsvendor model. The Newsvendor model, which is also named as a single-period model, assumes that while the sale is happening in an uncertain customer demand environment, unsold items are worthless at the end of the period. However, we do not discuss in this paper how an e-tailer sells these items at the end of period by using discounting

Table 2
Literature review.

Authors	Assortment planning type	Demand model	Substitution type	Decisions assortment option		Supplier effect	Chanel type	Basket effect	Method	Model enhancement
				Inventory	Pricing					
Ryzin and Mahajan [16]	Static	MNL	Static	YES	NO	NO	Offline	NO	Specialized heuristic	Developing MNL demand model for AP
Smith and Agrawal [18]		Exogenous	Dynamic	YES	NO	NO	Offline	NO	Specialized heuristic	Developing Exogenous demand model for AP
Cachon, et al. [20]	Static	MNL	Static	YES	NO	NO	Offline	NO	Iterative implementation of optimal assortment solution	Considering Consumer Search in AP Planning
Gaur and Honhon [17]	Static	locational demand model	Static	YES	NO	NO	Offline	NO	Specialized heuristic	Developing locational demand model for AP
Cachon and Kök [9]	Static	MNL	Static	YES	NO	NO	Offline	Yes	Game	Basket effect for AP / duopoly market structure
Maddah and Bish [24]	Static	MNL	Static	YES	YES	NO	Offline	NO	Specialized Heuristic	Pricing
Kök and Fisher [25]	Static	Exogenous	Dynamic	YES		NO	Offline	NO	Iterative Optimization Heuristic	Shelf-space limitation
Caro and Gallien [30]	Dynamic	Exogenous Demand with learning	Static	NO	NO	NO	Offline	NO	Lagrangian relaxation of dynamic programming	Dynamic assortment
Yücel, et al. [29]	Static	Exogenous	Static	YES	NO	YES, Supplier Selection	Offline	NO	Mixed-integer programming	Consider some operational issues such as shelf space constraint and supplier selection
Sauré and Zeevi [31]	Dynamic	General utility model	Static	NO	NO	NO	Both	NO	Special development heuristic for multi-armed bandit algorithm	Dynamic AP problem by considering capacity constraint
Davis, et al. [23]	Static	Nested logit	Static	NO	NO	NO	Offline	NO	Approximate pseudo-polynomial-time algorithm	Developing nested logit model
Besbes and Sauré [34]	Static	MNL	Static	NO	YES	NO	Offline	NO	Specialized heuristic	Different market structures
Bernstein and Martínez-de-Albéniz [41]	Dynamic	General Choice Models	Dynamic	NO	NO	NO	Offline	NO	Dynamic programming	Customized assortment for online retailing
Hübner, et al. [26]	Static	Exogenous Stochastic	Static	YES		NO	Offline	NO	Specialized heuristic	Perishable / Non-Perishable Products, Shelf Space Planning
Li, et al. [42]	Static	General Choice Models	Static	Yes	NO	Yes, Unknown Procurement Cost	Both	NO	Specialized heuristic	Procurement Contracting with Information Asymmetry, mechanism design, game theory model
Li, et al. [43]	Static	MNL	Static	Yes	No	No	Both	NO	Specialized heuristic	Comparing online and offline assortment decisions
Talebian, et al. [44]	Dynamic	Price-Response Function	Static	No	No	No	Offline	NO	Stochastic Dynamic Programming	Optimizing price with the purpose of Accelerating Demand Learning
Ghoniem, et al. [10]	Static	Utility model	Static	Yes	Yes	No	Offline	Yes	Mixed-integer nonlinear program	Pricing /cross-selling
My Work	**Static**	**MNL**	**Static**	**Yes**	**Yes**	**Yes, supplier Power**	**Online**	**Yes**	**Specialized heuristic**	**Game theory, Stackelberg model in assortment decisions**

mechanisms. Considering the fact that the most product assortments are planned seasonally, assuming the Newsvendor model in AP is logical. It should be also noted that if the replenishment lead-time is higher than the sale period, then it could happen at the first of that period otherwise the e-tailer is not able to purchase them.

We use the Multinomial logit model (MNL) and nested-MNL to model consumer choice decisions, which are a very common demand model in AP problems. Although MNL does not model stock-out based substitutions, it is a reasonable assumption in online shopping which e-tailer has lag time to fulfill customer demand from different fulfillment centers.

4. Problem notations and formulations

There are two categories of notations, which are used for modeling purposes, namely, parameters and decision variables. Tables 3–5 show the notations of these two groups along with their descriptions.

The leader decision about variable and fixed price of a given item assortment is denoted by two variables which are depicted in Table 4.

E-tailer follows supplier offers in this study, which forms its model decision variables. Three associated decision variables for e-tailer model are depicted in Table 5.

There are N differentiated products which the e-tailer can choose a set of variants of items to order to the supplier and shown to the customer for sale. We denote the chosen assortment with S which $S \subset N$. We use the Multinomial Logit model which has been widely used for formulating customer choice behavior in the literature [14]. The Multinomial Logit Model is a special type of Utility based model. In MNL, a utility value U_i is assigned for consuming each product variant which consists of deterministic and random parts:

$$U_i = u_i - \alpha p_i - \beta t + \xi_i \tag{1}$$

u_i can be interpreted as a maximum price which the customer is happy to pay, p_i is the i variant product price that α reflects the price sensitivity of customers on utility, t showing delivery lead time that β reflects the disutility of delivery lead time to customers' utility, and finally the random part that is modeled as an independent, identical Gumbel random variable reflexes the heterogeneity of customers' utility. In addition, U_0 is the utility of no shopping. So, each customer chooses the product in the offered assortment (S), which maximizes its utility along with no shopping decision. The probability of choosing item j in the available assortment in MNL model is equal:

$$\text{prob}_j(S) = \frac{\exp(U_j)}{\exp(U_0) + \sum_{j\in S}\exp(U_j)} \tag{2}$$

We refer to Anderson, et al. [45] for more details of the MNL model.

Without loss of generality, we consider two main types of customers based on the shopping attitudes. The first type of customers just consider the utility of the main item, although shop a basket of products. Chen et al. [12] named the main item as the lead category which attract customers for shopping. The second types of basket shopper customers consider the value of the total basket. As a consequence, the probability of shopping increases as they can find more desired items. Finally, we assume the number of customers who visit the store in the planning period, follows a Poisson random distribution with mean $\boldsymbol{\lambda}$. The probability that the customer is type 1 or 2 is $\boldsymbol{\lambda}_1$ and $\boldsymbol{\lambda}_2$ respectively.

We explained how the supplier and e-retailer associated objective functions and limitations are formed in the following subsections which is followed by the non-cooperative Stackelberg game model:

4.1. First strategy to decide about the main brand

4.1.1. The supplier model formulation

The supplier objective is to maximize its revenue, which is set as the supplier model's objective function. As supplier total revenue is dependent to two portions, namely, unit (variable) and the fixed price, the supplier should optimize its decisions for determining unit (variable) and fixed price that is charged to the e-tailer. These two prices are the decision variables in the supplier model. In summary, supplier total revenue is equal to: Purchase quantity of variant i by e-tailer* Unit (variable) price+ size of assortment * Fixed price, which is expressed as shown in Eq. (3):

$$max_{c,C}\left[\sum_{i\in S}(c * x_i)\right] + (C * |S|) \tag{3}$$

$$\textit{Subject to}:$$

$$c, C \geq 0 \tag{4}$$

As the model shows, the supplier profit is affected by the e-tailer decisions, which are reflected in x_i and S. Also, there is an assumption that the power of the supplier over the e-tailer is reflected in the limitations of the supplier model. The model limitations just limit the prices to be positive which does not impose any specific negative limitation over the supplier decision space. This model assumes that the supplier charges the e-tailer a fixed price for adding a new variety to the items assortment. This assumption is reflected in the second part of the objective function. Table 6 depicts the parts of the objective function along with an explanation of each variable used.

4.1.2. The E-tailer model formulation

The e-tailer's objective is to maximize its profit by optimizing its decisions that include assortment size and purchase quantity from the supplier besides the selling price to the customer. The selling price affects customer demand and total profit earned from it. It is evident that the e-tailer decisions are affected by the supplier decision on unit (variable) and fixed price as the e-tailer is following the supplier's decision. The e-tailer's total revenue is e-tailer profit = sales profit - purchase cost - delivery cost – holding cost, which is expressed in Eq. (5):

$$max_{S\subset N,X,p,r}\sum_{i\in S}\left[\left(p_i + \frac{(\lambda_1 r_{b1} + \lambda_2 r_{b2})}{(\lambda_1+\lambda_2)} - c - g(t)\right) * min(x_i, D_i)\right]$$
$$- [h * max(x_i - D_i, 0)] - [|S| * C] \tag{5}$$

$$\textit{Subject to}: d_i = \frac{e^{U(i)}}{\sum_{\forall j\in S} e^{U(j)} + e^{U(0)}} * (\lambda_1+\lambda_2) \tag{6}$$

$$x_i \geq Q, \forall i \in S \tag{7}$$

$$Q, p, r, t \geq 0 \tag{8}$$

Constraint (6) shows the total demand for item of variant i. Constraint (7) shows the minimum purchase quantity, which is imposed by the supplier. Table 7 shows different parts of the e-tailer model's objective functions an explanation of each.

In this model, we assume that we know the utilities of the e-tailer's customers for various items along with value of λ.

4.1.3. The non-cooperative Stackelberg game model

In this section, we model the interaction of the supplier and the e-tailer using Stackelberg game formulation. In this non-cooperative hierarchical game structure, one of the players, the leader, has more power to enforce its strategy on the other player(s), the follower. The solution concept is developed through

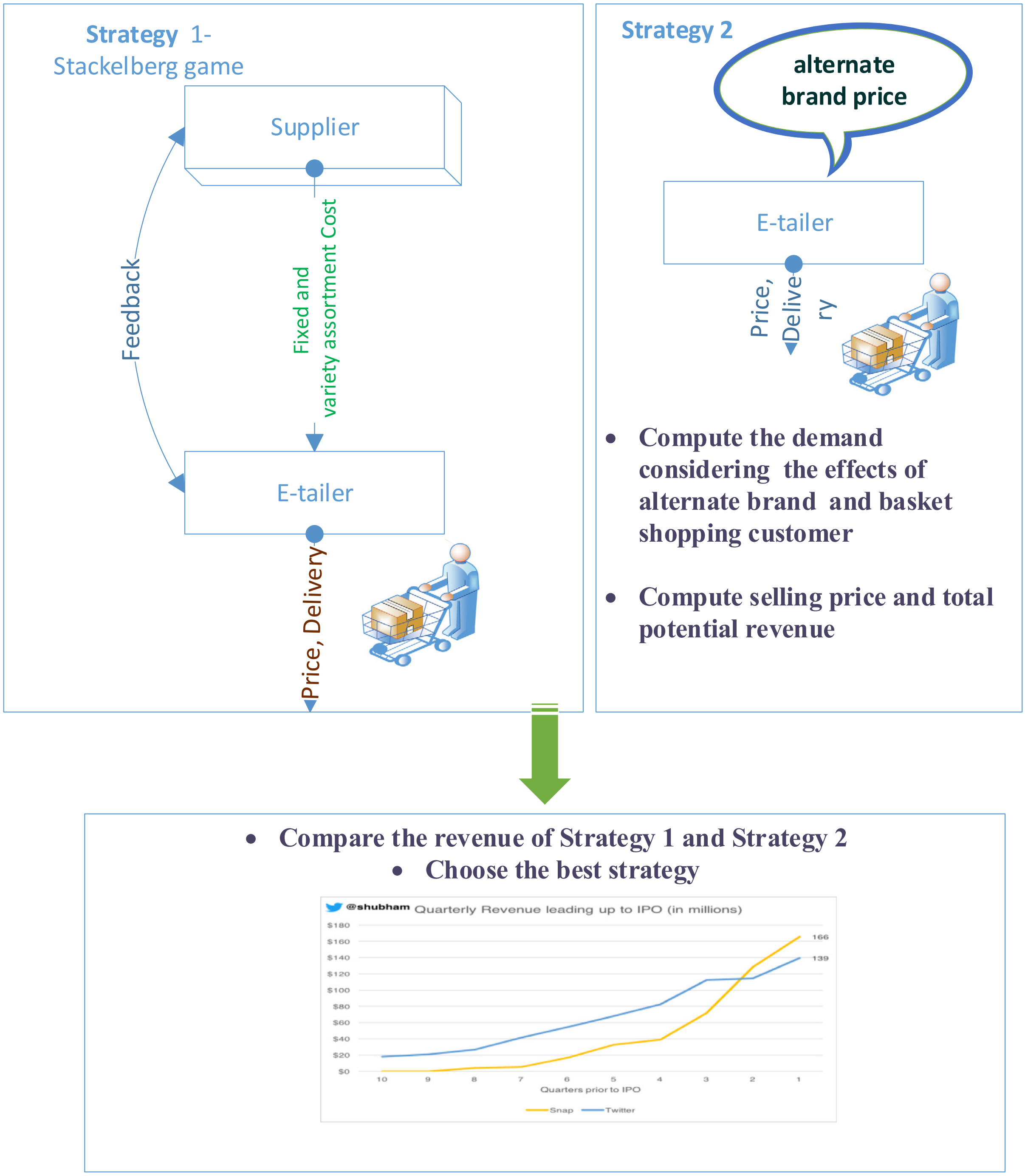


Fig. 1. E-tailer procurement decisions in a non-cooperative supply chain structure.

Table 3
Parameters notations and their descriptions.

Notations	Description	Related party(s)
$N = \{1, 2, \ldots, n\}, i \in N$	The set of different variant of items	Supplier, E-tailer
$\{u_1, u_2, \ldots, u_n\}$	items utility, $u_1 \geq u_2 \geq \cdots \geq u_n$	E-tailer
h	*holdingcost*	E-tailer
λ	Demand rate: Mean number of customers visiting the E-tailer's website per period	E-tailer
λ_1	The probability that the upcoming customer is from type 1	E-tailer
λ_2	The probability that the upcoming customer is from type 2	E-tailer
Q	Supplier minimum purchase quantity	Supplier
r_{b1}	Profit of Basket exclude main product for the customers in type 1	E-tailer
r_{b2}	Profit of Basket exclude main product for the customers in type2	E-tailer
$g(t)$	Delivery cost as a function of delivery time which is paid by the e-tailer	E-tailer

Table 4
Decision variables and their descriptions (Upper level (supplier)).

Notations	Description	Related party(s)
c	The unit (variable) price charged by supplier to the e-tailer	Supplier
C	Fixed price charged by the supplier to the e-tailer	Supplier

Table 5
Decision variables and their descriptions (Lower level (e-tailer)).

Notations	Description	Related party(s)
$S \subset N, j \in S$	Assortment decision	E-tailer
p_i	Price: Selling price of variant i	E-tailer
x_i	Purchase quantity of variant i	E-tailer
t	Delivery service	E-tailer

a hierarchal procedure. The first move is done by the leader, and the followers informedly react to this decision by playing their best move. In short, the leader optimizes its policy by considering the rational reaction of follower to its decision.

A Joint Pricing and Assortment Optimizing for Online Retailers model formulation for optimizing AP under Stackelberg game is as follows:

$$maxF_{x,S}\,(c, C) = \sum_{i\in S}(c * x_i) + (C * |S|) \tag{9}$$

Subject to :

$$c, C \geq 0 \tag{10}$$

$$maxf_{c,C}\,(S \subset N, X, p, r) = \sum_{i\in S}(p_i + (\lambda_1 r_{b1} + \lambda_2 r_{b2}) - c$$
$$- g\,(t)) * min(x_i, D_i) - h * max(x_i - D_i, 0) - |S| * C \tag{11}$$

Subject to :

$$x_i \geq Q \tag{12}$$

$$d_i = \frac{e^{U(i)}}{\sum_{\forall j\in S} e^{U(j)} + e^{U(0)}} * \lambda \tag{13}$$

$$Q, p, r, t \geq 0 \tag{14}$$

In this condition, the supplier as a leader seeks to maximize its revenue equation (9) while expecting the e-tailer to optimize its decisions. The supplier decisions directly influence the follower objective functions, which is the e-tailer profit over bought assortment (11). In optimizing follower model, the leader decision is a constant parameter. For a given supplier, decisions c (unit cost) and C (variable cost), the e-tailer determines its best decision through (11) -(14).

4.2. Second strategy to consider the alternate brand in modeling

The second strategy is optimizing assortment decisions for carrying an alternative brand. The developed model technically is similar to model part 'B'. However, the demand model should be revised to consider the affect of quality degradation as well as the effect of main brand utility in customer's minds. Objective function maximizes the e-tailer's profit by optimizing assortment size, inventory level and the selling price to the customer about alternate brand.

$$max_{S\subset N,X,p,r} \sum_{i\in S}\left[(p_i + \overline{r_b} - c - g(t)) * min(x_i, D_i)\right]$$
$$- [h * max(x_i - D_i, 0)] - [|S| * C] \tag{15}$$

$$\overline{r_b} = \lambda_1 * \frac{\sum_{\forall j\in S} e^{U'(j)}}{\sum_{\forall j\in S} e^{U(j)} + e^{U(0)} + \sum_{\forall j\in S} e^{U'(j)}} * r_{b1}$$
$$+ \lambda_2 * \frac{\sum_{\forall j\in B-A} r_j}{\sum_{\forall j\in B} r_j} * r_{b2} \tag{16}$$

$$Subject\ to: d'_i = \frac{\sum_{\forall j\in S} e^{U'(j)}}{\sum_{\forall j\in S} e^{U(j)} + e^{U(0)} + \sum_{\forall j\in S} e^{U'(j)}}$$
$$\times * \frac{e^{U'(j)}}{\sum_{\forall j\in S} e^{U'(j)} + e^{U(0)}}(\lambda_1) + \frac{\sum_{\forall j\in B-A} r_j}{\sum_{\forall j\in B} r_j} * \frac{e^{U'(j)}}{\sum_{\forall j\in S} e^{U'(j)} + e^{U(0)}}(\lambda_2) \tag{17}$$

$$x_i \geq Q, \forall i \in S \tag{18}$$

$$Q, p, r, t \geq 0 \tag{19}$$

The e-tailer profit is computed in Eq. (15), which is: sales profit - purchase cost - delivery cost – holding cost. Eq. (17) shows the demand computation for alternative brands. According to the Chen et al. [12], the shopping probability of the basket for the customers type 1 is equal to selling the main one. According to Cachon and Kok [9], we use the nested-MNL model to compute the probability of choosing an alternate brand as:

$$\frac{\sum_{\forall j\in S} e^{U'(j)}}{\sum_{\forall j\in S} e^{U(j)} + e^{U(0)} + \sum_{\forall j\in S} e^{U'(j)}} \tag{20}$$

And the probability of choosing variant i is:

$$\frac{e^{U'(j)}}{\sum_{\forall j\in S} e^{U'(j)} + e^{U(0)}} \tag{21}$$

So the type 1 customer demand is equal as

$$\frac{\sum_{\forall j\in S} e^{U'(j)}}{\sum_{\forall j\in S} e^{U(j)} + e^{U(0)} + \sum_{\forall j\in S} e^{U'(j)}} * \frac{e^{U'(j)}}{\sum_{\forall j\in S} e^{U'(j)} + e^{U(0)}}(\lambda_1) \tag{22}$$

For the type 2 customers, we should consider the utility of the whole basket. We use the approach of Mani, et al. [11] to compute probability of shopping or basket retention as they termed.

$$p_b^{ret} = pr\,\{basket\ B\ is\ retained\} = \frac{\sum_{\forall j\in B-A} r_j + \rho r_A}{\sum_{\forall j\in B} r_j} \tag{23}$$

$\sum_{\forall \boldsymbol{j}\in \boldsymbol{B}} \boldsymbol{r_j}$ is the total revenue generated from customers who shop the main item in basket B. As mentioned later, we consider one kind of basket for customers of type 2. However, the model can easily extend to more sub-types of each customer groups. ρ is a parameter which normalizes the effect of main product assortment on the probability of shopping basket that closer to 1 reflecting the customer satisfaction for a substitute and near to 0 means they are unhappy.

5. Solution procedure

Stackelberg game equilibrium solution is usually obtained by using a backward induction. It means that first the follower's problem, here the e-tailer, must be solved to find the response functions for the leader's (supplier) decisions. Next, the leader's optimum decision is found by considering the reactions of the follower from the previous response functions. This equilibrium solution is obtained by the assumption of rational decision making. Thus, each agent reacts by playing the best possible move. It should be mentioned, the follower's best response function is obtained for every possible leader moves. This issue can make the computation of equilibrium points so complex especially in cases

Table 6
Supplier model's objective function's part along with explanation.

Objective function's part	Explanation
$c * x_i$	Supplier revenue which is earned by selling variant i to e-tailer
$(C * \|S\|)$	Supplier revenue which is earned by the fixed cost that the supplier charges the e-tailer
$\sum_{i \in S}(c * x_i)$	Supplier revenue which is earned by Total variable cost that the supplier charges the e-tailer

Table 7
E-tailer model's objective function's part along with explanation.

Objective function's part	Explanation
$(p_i + \frac{(\lambda_1 r_{b1} + \lambda_2 r_{b2})}{(\lambda_1 + \lambda_2)} - c - g(t))$	E-tailer profit when selling an ith item
$min(x_i, D_i)$	Number of sold ith item
$max(x_i - D_i, 0)$	Number of leftovers for ith assortment in a given selling period
$h * max(x_i - D_i, 0)$	Cost of holding the leftover numbers of ith assortment

Table 8
Verifying the model on five facts and a suggestion.

#	Statement	Associated figures
Fact 1	By decreasing the alternate item's purchasing cost, e-tailer utility on main brand is decreased.	Fig. 3
Fact 2	By considering the profit of basket as a parameter in the model, the e-tailer is happy to pay more for purchasing the main brand if the associated basket creates profit for it.	Fig. 4
Fact 3	If the profit of associated basket of the main item is more than the expected value, the e-tailer considers less marginal profit on the main item to maintain basket customers.	Fig. 5
Fact 4	The distribution of two customer types affects the final procurement decision of the e-tailer	Fig. 6
Fact 5	Higher price sensitivity of demand leads to a lower profit for e-tailer.	Fig. 7
Suggestion	Model suggests bringing more assortments in the case of high Price sensitivity.	Fig. 7

where it is not possible to obtain the follower's best response analytically like in this paper. A classical approach for solving Bi-level programming is reduction to a single level using an approach like Karush--Kuhn--Tuckerconditions or a decent method. However, this approach is applicable for nice behavior models like a linear convex mathematical model. With the attention to integer or non-linear follower models, this approach is not applicable for assortment problem due to integer variable. Other techniques including exacts and evolutionary algorithm is developed in the literature based on the problem structures. With the attention to our problem structures, we use grid search to find the equilibrium solution. By attention to the point that fixed and variable cost is to be known with the attention to the market, we assume there is a known set for fixed and variable cost, which is based on the market condition. For each possible combination of these values, the lower problem is solved. Finally, the upper level profit is computed and the optimal solution is obtained.

We refer to Li, et al. [43], Maddah and Bish [24], and Smith and Agrawal [18] for good explanations of pricing and assortment planning problem solving procedures. With the assumption of N identical variant (N differentiation of identical products) of products, the solution is presented. This is a common assumption in most assortment planning studies, which leads to considering unique selling price and procuring cost. Also, for the ease of computation, the demand rate is approximated by using normal distribution with mean $\boldsymbol{\lambda} * \mathrm{prob}_j$ and variance of $\sqrt{\boldsymbol{\lambda} prob_j}$ which is a fairly accurate approximation [16,24]. By fixing the price, as Smith and Agrawal [18] discuss, the optimal inventory level is found as the same as the Newsvendor problem which equal as:

$$x^{**} = \lambda q_r + z(p)\sqrt{\lambda q_r} \tag{24}$$

$z(p) = \Phi^{-1}(\frac{p-c}{p-c+h})$ that $\Phi(.)$ is Standard Normal cumulative function. We have a minimum level order constraint imposed by the supplier. With the attention to the convexity of the Newsvendor problem we can write:

$$x^* = min(Q, x^{**}) \tag{25}$$

And the optimal service level is:

$$g\left(t^*\right) = -\beta \tag{26}$$

The profit function can be written as the function of price (p) and number of assortment (n) as:

$$f\,(n,p) = n * (p + (\lambda_1 r_{b1} + \lambda_2 r_{b2}) - c - g\,(t)) * \min\,(x, D) - h * \max\,(x - D, 0) - n * C \tag{27}$$

The optimal price level is:

$$p^*(n) = arg_p\{\frac{\partial f\;(n,p)}{\partial p} = 0\} \tag{28}$$

we should evaluate $f\,(n,p)$ for N possible assortment variant by Eqs. (24)–(28). Then the optimal assortment size is:

$$n^* = arg\{f\;(n,p)\} \tag{29}$$

The explained algorithm for Stackelberg game model is shown in Fig. 2.

6. Using game theory to achieve equilibrium of the problem

In this section, we demonstrate the obtained result from the developed model and assess if they align with the behavior expected from rational players and entities. The players and entities in the developed model are suppliers of the main brand, customers and e-tailer. The output from the model checked in their alignment with the five facts and suggestion shown in Table 8.

```
Algorithm 1. Stackelberg (C,c)
Input: set of feasible fixed costs: feasible_fixedCost,
       set of feasible variable costs: feasible_ variableCost
Output: Optimized fixed and variable costs: C, c
i, j ← 0
for C ∈ feasible_fixedCost
    j ← j + 1
    for c ∈ feasible_varibleCost
        [S, P, X] ← followersub_problem /Section IV.1.B
        i ← i + 1
        Supplier_profit[i, j] ← leadersub_problem(S, X) /Section IV.1.A
    end
end
[t] ← Argmax(supplier_ profit[i, : ])
[h] ← Argmax(supplier_ profit[: , j])
[C, c] ← feasible_fixedCost_t, feasible_varibleCost_h
Return Optimized fixed and variable costs: C, c
```

Fig. 2. Algorithm for Stackelberg game equilibrium solution.

Table 9
Value for initialing the required parameters.

Parameters	Values
N	6
$\{\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_6\}$	[5, 5, 5, 5, 5, 5]
$\boldsymbol{u}_0$	0.99
h	0.1*c
λ, Q	100,10

For running the proposed model, it needs to be tuned with its initial parameters. We use the following values shown in Table 9 for the required variables.

The procedure for finding the game equilibrium point is elaborated in Tables 10–12. Table 11 shows that for each possible supplier decision on the combination of fixed and variables costs, the best response function for the e-tailer in terms (assortment size, price, quantity). Supplier profit is computed by considering the e-tailer's reaction to its decisions. Finally, as shown in Table 10, the decision that brings maximum profit for the supplier is selected. By knowing the supplier offers about assortment cost, the e-tailer reaction and the corresponding profit are found out as shown in Tables 11–12. Table 11 shows the decision variables for the e-tailer in terms of (assortment size, price, quantity) whereas Table 12 shows the profit of the e-tailer for that combination of fixed and variable cost.

6.1. Using the game theory model on Fact 1

The objective of Fact 1 is to show the tradeoff between the procurement cost of the alternative brand and utility of the main item:

Fact 1. *By decreasing the alternative item's purchasing cost, E-tailer's utility on main item is decreased.*

The utility of an alternative brand is considered as the percent of the main brand's utility. For the e-tailer to decide the profitable procurement policy, the profit of the two strategies i.e. buying the main or the alternative brand are compared in Fig. 3. Furthermore, this figure shows that the e-tailer accepts more utility reduction in the alternative brand as the purchasing cost decreases. As shown in Fig. 3, when the utility of the alternative brand is 80% of the main brand and its cost is 17% of the main brand, the e-tailer would gain more profit by selling the alternative brand as compared to the main brand. In fact, we can make a more general rule by looking at the figure:

If the utility **of the alternative brand is in (0.6,1]** and **cost is 17 percent** of the main brand THEN **alternative brand** should be purchased.

When cost of buying alternative item is increased the rule which is inferred from the figure is as follows:

If utility **of alternative brand is in (0.8,1]** and **cost is 33 percent** of the main brand THEN **alternative brand** should be purchased.

6.2. Using the game theory model on Fact 2

Fact 2 examines the effect of considering basket profit for the e-tailer to decide which item to buy.

Fact 2. *By considering the profit of basket as a parameter in the model, E-tailer is happy to pay more for the item with the main brand as long as the associated basket creates profit for him.*

For showing the effect of considering basket shopping of the consumer on the procurement policy, we solve the problem by setting parameters in two different settings. In the first setting, the profit of the customer's basket shopping is considered as shown in Fig. 4(a) while in Fig. 4(b), we solve the model by just considering the item's demand and profit without the basket effect. The graphs show that by comparing its profit in these two settings, the e-tailer can decide which procurement policy to choose. As Fig. 4(a) shows, considering basket revenue in modeling, the e-tailer should buy the alternative brand if, and only if, its variable cost is less than $0.5. However, if the basket profit is not considered in modeling Fig. 4(b) then even with an alternative brand's variable cost of $1, the model recommends

Table 10
Leader (Supplier) profit function.

Supplier objective function		Fixed cost (C $)					
		0	10	20	30	40	50
Variable cost(c $)	1	67.3701	101.2426	113.2819	133.2819	115.0924	125.0924
	1.5	107.123	150.7868	154.2225	174.2225	194.2225	164.8799
	2	151.763	203.2186	215.6907	216.2106	236.2106	256.2106
	2.5	198.8193	258.8193	280	289.9174	278.6723	298.6723
	3	247.8287	307.8287	**347.4845**	334.8238	321.1372	341.1372

Table 11
Follower (E-tailer) Best response of (assortment size, selling price, quantity) decisions.

Best response of (assortment size, selling price, quantity) decisions of the e-tailer							
		Fixed cost ($C)					
		0	10	20	30	40	50
Variable cost ($c)	1	(5, 4.92, 13.47)	(3, 4.49, 23.75)	(2, 4.16, 36.64)	(2, 4.16, 36.64)	(1, 3.59, 75.09)	(1, 3.59, 75.09)
	1.5	(6, 5.15, 11.90)	(4, 4.81, 18.46)	(2, 4.25, 38.07)	(2, 4.25, 38.07)	(2, 4.25, 38.07)	(1, 3.69, 76.59)
	2	(6, 5.24, 12.65)	(5, 5.09, 15.32)	(3, 4.67, 25.95)	(2, 4.35, 39.05)	(2, 4.35, 39.05)	(2, 4.35, 39.05)
	2.5	(6, 5.33, 13.25)	(6, 5.33, 13.25)	(4, 5.00, 20.00)	(3, 4.77, 26.66)	(2, 4.45, 39.73)	(2, 4.45, 39.73)
	3	(6, 5.42, 13.77)	(6, 5.42, 13.77)	**(5, 5.28, 16.50)**	(3, 4.88, 27.20)	(2, 4.57, 40.19)	(2, 4.57, 40.19)

Table 12
Follower (E-tailer) profit function; e-tailer objective function.

Variable cost ($c)	Fixed cost ($C)					
	0	10	20	30	40	50
1	429.834	394.017	371.545	351.545	339.361	329.361
1.5	436.567	386.558	356.558	336.558	316.558	306.449
2	432.314	374.96	339.449	315.440	295.44	275.44
2.5	420.516	360.516	316.86	279.389	252.716	232.716
3	371.827	311.827	**254.376**	218.412	194.496	174.496

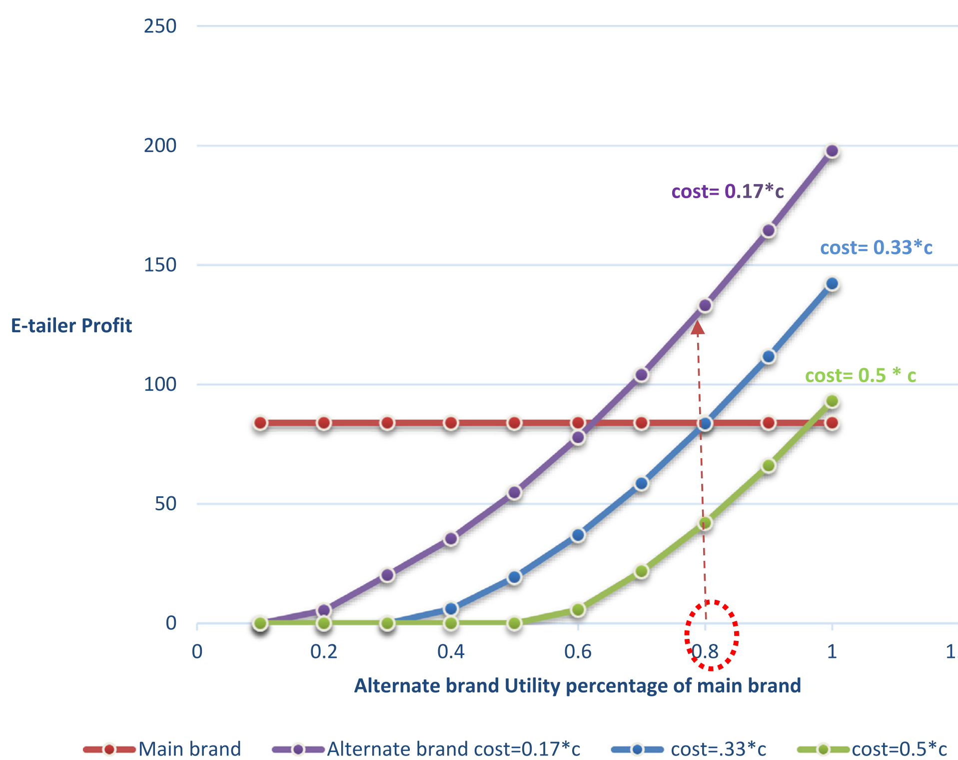
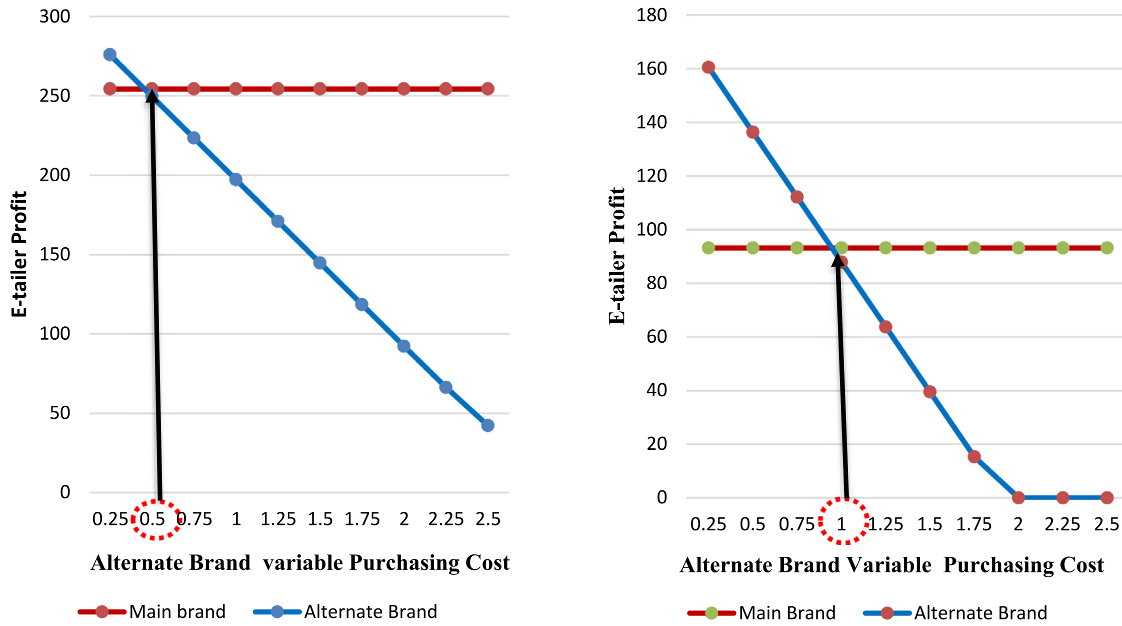


Fig. 3. Analysis of tradeoff between cost and utility of the alternative brand.

it buy the alternative brand. In this scenario, while the e-tailer pays more, it still gets the alternative brand because the model did not consider the basket profit. This shows the importance of considering basket shopping profit in the modeling.

6.3. *Verifying model on Fact 3*

This fact examines the tradeoff between basket profit and the main brand profit.



(a) (b)

Fig. 4. Analysis of (a) considering and (b) not considering basket profit in choosing procurement strategies.

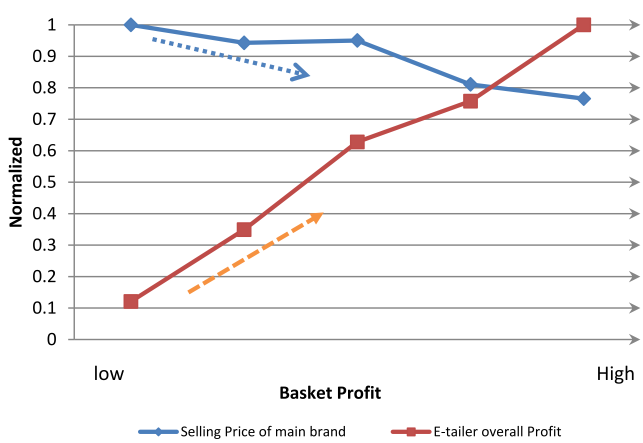


Fig. 5. Analysis of basket profit on price as e-tailer decision variables.

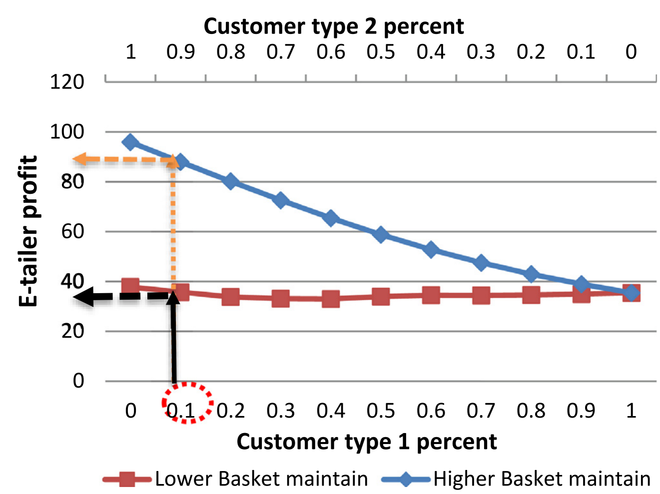


Fig. 6. Analysis of different type of customer.

Fact 3. *If the profit of associated basket with the main item is more than the expected value, e-tailer considers less marginal profit on the main item to maintain basket customers.*

In this section, we investigate the basket profit effect on the e-tailer profit and decision variables. E-tailer profit and selling price are normalized for them to be shown in one graph. As Fig. 5 shows, the higher the profit of the basket with the main item, the e-tailer is happy to reduce the selling price of the item to retain its high demand which finally leads to higher profit. It shows that by considering basket profit, it could change the e-tailer decision variables in assortment planning problem. Fig. 5 also shows that as the selling price decreases, the e-tailer's profit is increasing as basket profit is increased. It is interesting to see that the gradient in the figure of the e-tailer's profit is much steeper in increasing rather than decreasing in selling price which results in an increased basket profit.

6.4. Verifying model on Fact 4

In this fact, we examine how the distribution of two types of customer affects the final procurement decision of the e-tailer.

Fact 4. *The distribution of two customer types affects the final procurement decision of the e-tailer.*

As mentioned, we consider two types of customer. As Table 3 shows, the first type of customer just considers the utility of the lead item while the second one considers the whole basket utility. We denote the percent of first and the second types of customers with L_1 and L_2 respectively in the analysis.

Let us assume, as an example, that the utility of the basket for the second type of customer on one occasion is 0.33 and on

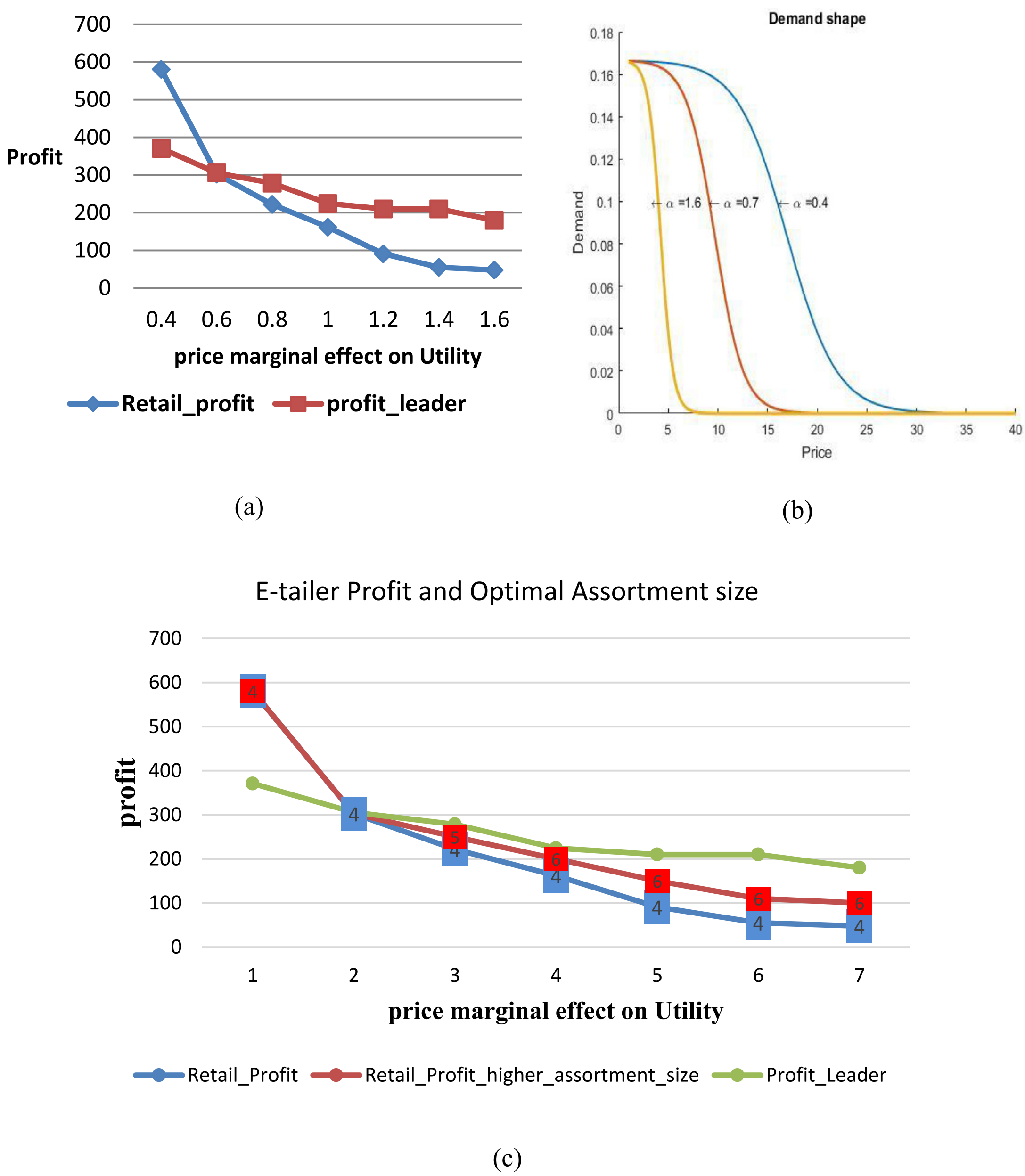


Fig. 7. Analysis of price sensitivity in demand model and its affect on supplier and e-tailer profit and assortment size.

another occasion it is 0.66, denoted by red and blue lines respectively of Fig. 6. Fig. 6 depicts the e-tailer's profit over various percent of customer types. For example, when an e-tailer has 10 percent type one customer (90% type two), its profit for the two settings are very different. When the basket utility is 0.66 then the e-tailer is getting more profit. However, this is not true when the utility is 0.33. This is an acceptable fact as the probability of purchasing an item has increased when the customer places more importance on the basket and the item is a part of that basket. This can lead to the managerial insight that if the attractiveness of the basket of the main item is high for customer type 2, and customer type two is more than type one, then the e-tailer has more flexibility whether it procures the lead item from the main or alternative brand.

According to the figure, if the probability of maintaining the basket is high, the e-tailer has more flexibility in decision making.

6.5. *Verifying model on Fact 5*

This fact states that when the demand is greatly dependent on the price, the e-tailer's profit decreases.

Fact 5. *Higher value of price impacts on customer utility thereby leading to a lower profit.*

As Fig. 7(b) shows the rate of drop of customer demand in a product is less steep when alpha is 0.4 in comparison to when alpha is 1.6. Thus, if the demand shape is like the blue line (Fig. 7(b)), the e-tailer gets more profit as an increasing price increases its profit while not losing many customers. Also, Fig. 7(c) shows how the e-tailer is able to increase its profit by providing a wider assortment size when the price sensitivity is increasing. In brief, when the *Price marginal effect* is increased which led to the e-tailer profit decrement, the e-tailer should provide a wider assortment.

Summary: In summary, analyzing different cases shows that considering basket revenue in assortment planning can lead to selecting different policies in procurement. Examining different scenarios in basket shopping habits can better guide the e-tailer in selecting its supplier. If the item plays an important role in maintain the shopping basket, it is worth paying more to the supplier to having a higher quality or a customized product. If the basket containing the item has enough retention for shopping by the customer, or the substituted alternative brand has enough utility for the customer compared to the main one, the e-tailer has greater flexibility in supplying the item.

7. Conclusion

In this paper, we propose a game-theory model to determine an assortment planning policy for the e-tailer. We extend the literature in two areas of AP: supplier effect and basket shopping consumer. We assume the e-tailer has two strategies in offering product assortments. It can provide products with supreme quality by the main supplier or just simply offer alternative products. In the first strategy, we use a non-cooperative Stackelberg game model as the supplier has more power over the e-tailer. By analyzing different scenarios, we conclude that the product assortment decisions including assortment size, selling price to the customer, and main or alternative brand is dependent on some main factors like the role of the item in the retaining basket, product demand parameters, and utility level of the main and alternative brands. The e-tailer chooses to offer the main brand and interact with a powerful supplier in cases the item is playing the main role in attracting purchasing by the customer or the basket revenue is high enough. However, the e-tailer prefers to offer an alternative brand when the item has low impact in basket retention or the alternative brand has acceptable utility and purchasing cost. The proposed paper is considered the interaction of the e-tailer with powerful the supplier. The proposed model considers the interaction between e-tailer and powerful supplier. For the future work, researchers can extend the structure of competition by adding a bricks and mortar retailer as the third player and find how the new structure work under various scenarios.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] C. Hart, M. Rafiq, The dimensions of assortment: A proposed hierarchy of assortment decision making, Int. Rev. Retail, Distrib. Consum. Res. 16 (3) (2006) 333–351.

[2] M.K. Mantrala, M. Levy, B.E. Kahn, E.J. Fox, P. Gaidarev, B. Dankworth, D. Shah, Why is assortment planning so difficult for retailers? A framework and research agenda, J. Retail. 85 (1) (2009) 71–83.

[3] N. Flannigan, Economic Structure and Performance of the Australian Retail Industry, Productivity Commission Draft Report, 2011.

[4] M.-H. Hsiao, Shopping mode choice: Physical store shopping versus e-shopping, Transp. Res. Part E: Logist. Transp. Rev. 45 (1) (2009) 86–95.

[5] P.N. Bloom, V.G. Perry, Retailer power and supplier welfare: The case of Wal-Mart, J. Retail. 77 (3) (2001) 379–396.

[6] C. Phillips, Online retailers suffer at the hands of suppliers, 2012, Available: http://www.powerretail.com.au/pureplay/online-retailers-suffer-suppliers/. (Accessed 27 January 2012).

[7] N. Kumar, The power of trust in manufacturer-retailer relationships, Harv. Bus. Rev. 74 (6) (1996) 92.

[8] K. Kwon, T. Cheong, A minimax distribution-free procedure for a newsvendor problem with free shipping, European J. Oper. Res. 232 (1) (2014) 234–240.

[9] G.P. Cachon, A.G. Kök, Category management and coordination in retail assortment planning in the presence of basket shopping consumers, Manage. Sci. 53 (6) (2007) 934–951.

[10] A. Ghoniem, B. Maddah, A. Ibrahim, Optimizing assortment and pricing of multiple retail categories with cross-selling, J. Global Optim. 66 (2) (2016) 291–309.

[11] V. Mani, D.J. Thomas, S. Bansal, Estimating substitution and basket effects in retail stores: Implications for assortment planning [Online]. Available: https://ssrn.com/abstract=2878215.

[12] Y. Chen, J.D. Hess, R.T. Wilcox, Z.J. Zhang, Accounting profits versus marketing profits: A relevant metric for category management, Mark. Sci. 18 (3) (1999) 208–229.

[13] Z. Saberi, O. Hussain, M. Saberi, E. Chang, Online retailer assortment planning and managing under customer and supplier uncertainty effects using internal and external data, in: 2017 IEEE 14th International Conference on e-Business Engineering, ICEBE, IEEE, 2017, pp. 7–14.

[14] A.G. Kök, M.L. Fisher, R. Vaidyanathan, Assortment planning: Review of literature and industry practice, in: Retail Supply Chain Management, Springer, 2008, pp. 99–153.

[15] M. Deb, Assortment Planning of Vertically Differentiated Products under Consumer Choice (Doctor of Philosophy, Business Administration), Pennsylvania State University, 2013.

[16] G. v. Ryzin, S. Mahajan, On the relationship between inventory costs and variety benefits in retail assortments, Manage. Sci. 45 (11) (1999) 1496–1509.

[17] V. Gaur, D. Honhon, Assortment planning and inventory decisions under a locational choice model, Manage. Sci. 52 (10) (2006) 1528–1543.

[18] S.A. Smith, N. Agrawal, Management of multi-item retail inventory systems with demand substitution, Oper. Res. 48 (1) (2000) 50–64.

[19] S. Mahajan, G. Van Ryzin, Stocking retail assortments under dynamic consumer substitution, Oper. Res. 49 (3) (2001) 334–351.

[20] G.P. Cachon, C. Terwiesch, Y. Xu, Retail assortment planning in the presence of consumer search, Manuf. Serv. Oper. Manag. 7 (4) (2005) 330–346.

[21] C. d'Aspremont, J.J. Gabszewicz, J.-F. Thisse, On hotelling's stability in competition, Econometrica (1979) 1145–1150.

[22] K. Lancaster, Socially optimal product differentiation, Am. Econ. Rev. 65 (4) (1975) 567–585.

[23] J.M. Davis, G. Gallego, H. Topaloglu, Assortment optimization under variants of the nested logit model, Oper. Res. 62 (2) (2014) 250–273.

[24] B. Maddah, E.K. Bish, Joint pricing assortment and inventory decisions for a retailer's product line, Nav. Res. Logist. 54 (3) (2007) 315–330.

[25] A.G. Kök, M.L. Fisher, Demand estimation and assortment optimization under substitution: Methodology and application, Oper. Res. 55 (6) (2007) 1001–1021.

[26] A. Hübner, H. Kuhn, S. Kühn, An efficient algorithm for capacitated assortment planning with stochastic demand and substitution, European J. Oper. Res. 250 (2) (2016) 505–520.

[27] A. Hübner, A decision support system for retail assortment planning, Int. J. Retail Distrib. Manage. 45 (7/8) (2017) 808–825.

[28] A. Hübner, K. Schaal, An integrated assortment and shelf-space optimization model with demand substitution and space-elasticity effects, European J. Oper. Res. 261 (1) (2017) 302–316.

[29] E. Yücel, F. Karaesmen, F.S. Salman, M. Türkay, Optimizing product assortment under customer-driven demand substitution, European J. Oper. Res. 199 (3) (2009) 759–768.

[30] F. Caro, J. Gallien, Dynamic assortment with demand learning for seasonal consumer goods, Manage. Sci. 53 (2) (2007) 276–292.

[31] D. Sauré, A. Zeevi, Optimal dynamic assortment planning with demand learning, Manuf. Serv. Oper. Manag. 15 (3) (2013) 387–404.

[32] F. Bernstein, A.G. Kök, L. Xie, Dynamic assortment customization with limited inventories, Manuf. Serv. Oper. Manag. 17 (4) (2015) 538–553.

[33] N. Golrezaei, H. Nazerzadeh, P. Rusmevichientong, Real-time optimization of personalized assortments, Manage. Sci. 60 (6) (2014) 1532–1551.

[34] O. Besbes, D. Sauré, Product assortment and price competition under multinomial logit demand, Prod. Oper. Manage. 25 (1) (2016) 114–127.

[35] N. Agrawal, S.A. Smith, Optimal retail assortments for substitutable items purchased in sets, Nav. Res. Logist. 50 (7) (2003) 793–822.

[36] G.P. Cachon, S. Netessine, Game theory in supply chain analysis, in: Models, Methods, and Applications for Innovative Decision Making: INFORMS, 2006, pp. 200–233.

[37] X. He, A. Prasad, S.P. Sethi, G.J. Gutierrez, A survey of Stackelberg differential game models in supply and marketing channels, J. Syst. Sci. Syst. Eng. 16 (4) (2007) 385–413.

[38] M. Esmaeili, M.-B. Aryanezhad, P. Zeephongsekul, A game theory approach in seller–buyer supply chain, European J. Oper. Res. 195 (2) (2009) 442–448.

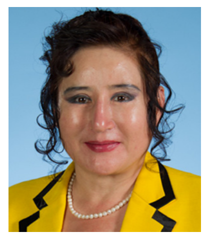
[39] S. Jørgensen, G. Zaccour, A survey of game-theoretic models of cooperative advertising, European J. Oper. Res. 237 (1) (2014) 1–14.

[40] D. Yue, F. You, Stackelberg-game-based modeling and optimization for supply chain design and operations: A mixed integer bilevel programming framework, Comput. Chem. Eng. 102 (2017) 81–95.
[41] F. Bernstein, V. Martínez-de Albéniz, Dynamic product rotation in the presence of strategic customers, Manage. Sci. (2016).
[42] Z. Li, L. Shao, D. Sun, Designing multi-attribute procurement mechanisms for assortment planning, Decis. Sci. 46 (3) (2015) 597–622.
[43] Z. Li, Q. Lu, M. Talebian, Online versus bricks-and-mortar retailing: A comparison of price, assortment and delivery time, Int. J. Prod. Res. 53 (13) (2015) 3823–3835.
[44] M. Talebian, N. Boland, M. Savelsbergh, Pricing to accelerate demand learning in dynamic assortment planning for perishable products, European J. Oper. Res. 237 (2) (2014) 555–565.
[45] S.P. Anderson, A. De Palma, J.F. Thisse, Discrete Choice Theory of Product Differentiation, MIT press, 1992.

Zahra Saberi is currently a Ph.D. candidate at School of Business, University of New South Wales (UNSW), Canberra. Prior to joining UNSW, she received her BSc in industrial engineering from the University of Sharif and master degree in Logistic and Supply chain engineering from the Amirkabir University. Her research interests mainly focus on E-tailing Operations Management, Retailing Analytics and Revenue Management. She was among the excellent student during the all education years and receives many prizes and certifications in these years.

Morteza Saberi is a Lecturer with School of Information, Systems and Modelling, University of Technology Sydney, Australia and has an outstanding research records and significant capabilities in the area of Business Intelligence, Data Mining and Applied Machine Learning. He has published More than 150 papers in reputable academic journals and conference proceedings. His Google Scholar citations and h-index are 2210 and 22 respectively. He was a Lecturer at the Department of Industrial Engineering at University of Tafresh. He is also the recipient of the 2006–2011 Best Researcher of Young Researchers Club, Islamic Azad University (Tafresh Branch). He is also the recipient of National Eminent Researcher Award among Young Researchers Club, Islamic Azad University members.

Omar Hussain is a senior lecturer at the University of New South Wales, Canberra. His research interests are in business intelligence, cloud computing and logistics informatics. In these areas, his research work focuses on utilizing decision making techniques for facilitating smart achievement of business outcomes. His research work has been published in various top international journals such as Information Systems, The Computer Journal, Knowledge Based Systems, Future Generation of Computer Systems etc. He has won awards and funding From competitive bodies such as the Australian Research Council for his research.



Elizabeth Chang is Professor and Canberra Fellow at the UNSW at the Australian Defence Force Academy (ADFA). She has 30 years of work experience in both Academia and Industry. She has been a full Professor in IT, Software Engineering and Logistics Informatics for 14 years. She had been in senior positions in commercial corporations for 10 years, typically working on commercial grade large software development. Her key research strength is in large complex software development methodologies, requirement engineering, structure and unstructured database design and implementation, trust, security, risk and privacy. In the 2012 edition of MIS Quarterly vol. 36 issue. 4 Special Issues on Business Research, Professor Chang was listed fifth in the world for researchers in Business Intelligence.