J Intell Manuf (2011) 22:505–521
DOI 10.1007/s10845-009-0312-8

Solving the multi-response problem in Taguchi method by benevolent formulation in DEA

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Received: 9 October 2008 / Accepted: 24 August 2009 / Published online: 16 September 2009

Abstract The Taguchi method is an efficient approach for optimizing a single quality response. In practice, however, most products/processes have more than one quality response of main interest. Recently, the multi-response problem in the Taguchi method has gained a considerable research attention. This research, therefore, proposes an efficient approach for solving the multi-response problem in the Taguchi method utilizing benevolent formulation in data envelopment analysis (DEA). Each experiment in Taguchi's orthogonal array (OA) is treated as a decision making unit (DMU) with multiple responses set inputs and/or outputs. Each DMU is evaluated by benevolent formulation. The ordinal value of the DUM's efficiency is then used to decide the optimal factor levels for multi-response problem. Three frequently-investigated case studies are adopted to illustrate the proposed approach. The computational results showed that the proposed approach provides the largest total anticipated improvement among principal component analysis (PCA), DEA based ranking approach (DEAR) and other techniques in literature. In conclusion, the proposed approach may provide a great assistant to practitioners for solving the multi-response problem in manufacturing applications on the Taguchi method.

Keywords Multi-response problem · Taguchi method · DEA · Benevolent formulation

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Introduction

Failure to select the best condition of process factors is a costly mistake in today's highly competitive markets. The overall goal of robust design is to find settings of the controllable factors so that the response is least sensitive to variations in the noise variables, while still yielding an acceptable mean level of the response. Taguchi (1991) method is a widely used approach for robust design, which utilizes an orthogonal array (OA) to obtain dependable information about the design parameter with minimum time and resources, and adopts signal-to-noise (S/N) ratio to interpret experimental data and optimize performance. Nevertheless, this method is found only efficient for determining the optimal setting of controllable factor levels which optimize a single response problem; such as, flank wear (Tsao and Hocheng 2002), thickness of solder paste (Li et al. 2008) and casting porosity (Anastasiou 2002).

Today's sharp market competition has forced most industries to produce products with more than one response. To solve the multi-response problem, the Taguchi method adopts a trade-off between quality loss and productivity by engineering judgment (Phadke 1989). But, this approach may provide contradictory optimal factor levels for multi-response problem and increases uncertainty in decision-making process. Shiau (1990) assigned a weight for each response then employed the sum of the weighted responses. Tong et al. (1997) used the S/N ratio for the sum of the weighted normalized quality losses. However, it still remains difficult to decide a weight for each quality response in real applications. Reddy et al. (1997) adopted regression techniques-based approaches to optimize the multi-response problem. Unfortunately, regression approaches increase the complexity of computational process and thus require statistical skills. Further, Antony (2000) utilized principal component analysis (PCA) to transform the multi-responses in few uncorrelated ones. The principal components were then used to find the optimal factor levels for multi-responses. But, PCA is based on some rigid assumption that the error terms are multivariate normally distributed random variable, which may limit its application in real applications. Furthermore, Yu et al. (2004) proposed a strategy called fuzzy neural-Taguchi network with genetic algorithm, Jeyapaul et al. (2006) deployed genetic algorithm, and Al-Refaie et al. (2008) employed grey analysis for optimizing the multi-response problem in the Taguchi method. In reality, the soft computing techniques genetic algorithm and neural networks, and grey analysis are still not be completely understood by many practitioners.

Data envelopment analysis (DEA) proposed by Charnes et al. (1978) is a fractional mathematical programming technique for evaluating the relative efficiency of homogeneous decision making units (DMUs) with multiple inputs and multiple outputs. DEA combines various inputs and various outputs for a DMU into one performance measure, called relative efficiency. DEA techniques can be divided into two categories. The techniques in the first category incorporate a priori information provided by a decision maker or expert, whereas the techniques in the other category do not incorporate a priori information (Angulo-Meza and Lins 2002). The first category includes direct weight restrictions (Dyson and Thanassoulis 1988), the cone ratio model (Charnes et al. 1990), and value efficiency analysis Halme et al. 2000). However, there are disadvantages in these methods concerning subjectivity. First, the priori information can be wrong or biased. Second, there may be a lack of consensus among the experts or decision-makers, which may adversely affect the study. In contrast, the techniques in the second category do not rely on priori information, provide reliable conclusions, and consequently, improve the decision making process. Among the techniques in the second category is benevolent formulation in cross-evaluation. Unlike the traditional DEA techniques, benevolent formulation increases discrimination among efficient DMUs and allows for DMU's peer-evaluation instead of self-evaluation. To utilize these advantages and reduce the uncertainty in the decision making process, this research employs benevolent formulation for solving the multi-response in the Taguchi method. Each experiment of Taguchi's array is treated as a DMU with multi-responses set as inputs and/or outputs. Benevolent formulation is then used to evaluate performance for each DMU and decide optimal factor levels for multi-responses. The remainder of this paper is organized as follows. DEA is introduced in "Data envelopment analysis". The proposed approach is presented in "The proposed approach". The application of the proposed approach is illustrated in "Application of the proposed approach". Conclusions are finally summarized in the last section.

Data envelopment analysis

Data envelopment analysis has been widely used for evaluating performance for a set of DMUs with multiple inputs and multiple outputs at organizational level, such as banks, hospitals, and universities (Charnes et al. 1994). The widely-used DEA technique is the CCR model, which is developed by Charnes et al. (1978).

The CCR model

The CCR model displayed in "Appendix A" measures the relative efficiency of each DMU once by comparing it to a group of the other DMUs that have the same set of inputs and outputs. Let DMU$_o$ denotes a DMU to be evaluated. Then, DMU$_o$ is identified as CCR-efficient if its relative efficiency, E_o, equals one. Otherwise, it is identified as inefficient. However, the CCR model fails to discriminate among efficient DMUs, since the relative efficiency scores may be equal to one for more than one DMU. On the other hand, Khouja (1995) adopted two phases approach in the selection of advanced manufacturing technology (robots) from a list of feasible technologies. In the first phase, the robot efficiencies are identified by the CCR model then evaluated by a multi-attribute decision-making model in the second phase. Liao (2005) used neural networks to predict the responses for all factor level combinations. The CCR model is then employed to decided optimal factor settings. However, Baker and Talluri (1997) investigated the robot selection problem in Khouja's study and showed that CCR model has an intrinsic problem that provides misleading efficiency scores through identifying a DMU with an unrealistic weighing scheme to be efficient. In addition, it may result in multiple optimal solutions. To avoid the above shortcoming of CCR model, this research utilizes the benevolent formulation in cross-evaluation to measure and compare performance of a set of DMUs.

Benevolent formulation in cross-evaluation

The cross-evaluation technique, initially introduced by Sexton et al. (1986), uses DEA in a peer-evaluation instead of a self-evaluation calculated by CCR model. A self-evaluation is to measure DMU$_o$'s efficiency is calculated using its own input and output weights. Whereas, a peer-evaluation means that DMU$_o$ is evaluated according to the optimal weighting scheme of other DMUs. The mean of these efficiencies is the cross-evaluation. However, multiple optimal solutions can exist, which cause the cross-efficiencies to vary. This problem is solved by introducing a secondary objective function using benevolent formulation. The main idea of benevolent formulation is to obtain a weighing scheme of DMU$_o$ that would be optimal in CCR model, but have, as a secondary objective, maximization of the cross-efficiencies of the other DMUs (Angulo-Meza and Lins 2002). This technique can be expressed by two models I and II shown in "Appendix B". By either model, once the optimal u_{ro} and v_{io} values, or u^*_{ro} and v^*_{io} respectively, are obtained, the cross-efficiencies of DMU$_o$ can be calculated. Let E_{oj} denotes the cross-efficiency of DMU$_j$ calculated according to the optimal weights of DMU$_o$. The E_{oj} is calculated as:

$$E_{oj} = \sum_{r=1}^{s} u^*_{ro} y_{rj} / \sum_{i=1}^{m} v^*_{io} x_{ij} \quad j \neq 0. \tag{1}$$

Once the E_{oj} values are calculated, a matrix called the "cross-efficiencies matrix" is constructed. Let e_j be the mean of cross-efficiencies for DMU$_j$ estimated as:

$$\mathrm{e}_j = \sum_{o \neq j} E_{oj}/(n-1) \quad j = 1, \ldots, n. \tag{2}$$

The e_j values are then used for comparing performance of n DMUs. Unlike the CCR model, the benevolent formulation increases discrimination among efficient DMUs by allowing efficiency takes a value greater than one. Benevolent formulation models are employed for solving the multi-response problem in the Taguchi method as described in the following section.

The proposed approach

Products have quality responses that describe their performance relative to customer requirements or expectations. Generally, a process/product quality characteristic (QCH) or response is divided into three main types: the smaller-the-better (STB), the nominal-the-best (NTB), and the larger-the-better (LTB) responses. The STB response has an ideal target of zero, such as electromagnetic radiations from telecommunications equipment and leakage current in integrated circuits. The NTB characteristic has a specific user-defined target value, such as the output voltage supply of television. Finally, the LTB response has an ideal state or target of infinity, such as the mechanical strength of a wire per unit cross-section area. In practice, the multi-responses of a product/process are not necessarily belonging to the same response type. In this research, it is assumed that the responses are uncorrelated. In these regards, the proposed approach for solving the multi-response problem in the Taguchi method is outlined in the following steps:

Step 1 Assume n experiments are conducted utilizing Taguchi's OA. Treat each experiment as a DMU. As mentioned earlier, the relative efficiency is defined as the sum of weighted outputs divided by the sum of the weighted inputs. Typically, higher efficiency indicates better performance, which can be achieved if the sum of the weighted outputs increases and/or the sum of the weighted inputs decreases. To enhance the relative efficiency of each DMU and achieve the desired target of each quality response, set the input and output of each DMU as follows:

(i) If all responses are STB type, then set all responses as inputs, whereas set a unit (one) as the output. Conversely, if all responses are LTB type, set all responses as outputs, while set a unit (one) as the input for all DMUs. In other words, the efficiency is improved by decreasing the denominator in the former case, or increasing the numerator in latter case.

(ii) If all responses are NTB type, then calculate the quality loss, L_j, for DMU$_j$ as follows (Tong et al. 1997):

$$L_j = c(s_j^2/\bar{y}_j^2) \quad j = 1, \ldots, n \tag{3}$$

where c is the quality loss coefficient, while $\bar{y}_j$ and s_j are the average and standard deviation of response replicates for of DMU$_j$, respectively. Since the objective is to minimize the quality loss, set the L_j values as inputs and one as the output for all DMUs.

(iii) If responses are a mix of the three types, set STB type response and L_j values of the NTB type response as inputs, whereas set LTB type response(s) as the output for all DMUs.

Step 2 Evaluate the relative efficiency, E_o, of each DMU by solving the input-oriented CCR model.

Step 3 Estimate the optimal input and output weights, u^*_{ro} and v^*_{io}, by solving Model I for each DMU$_o$. Then, calculate the cross-efficiency, E_{oj}, scores for each DMU using Eq. (1). Construct the cross-efficiencies matrix. Then obtain the average, e_j, of cross-efficiencies using Eq. (2).

Step 4 In order to optimize performance, and avoid the bias produced by large e_j values in selecting optimal levels, decide the ordinal value of e_j; the ordinal value is to rank the e_j values such that the smallest e_j value receives an ordinal value of one, whereas the largest e_j value takes an ordinal value of n. Let AOV$_{fl}$ be the average of the ordinal values for level l of factor f. Calculate the AOV$_{fl}$ value for each factor level. Typically, higher AOV$_{fl}$ implies better performance. Therefore, the optimal factor level, l^*, is identified as the level that maximizes the value of AOV$_{fl}$. Mathematically,

$$l^* = \left\{ l \,\middle|\, \max_{l} \{\mathrm{AOV}_{fl}\} \right\} \quad \forall f \tag{4}$$

If a tie occurs in selecting the optimal level for a factor, choose the factor level that provides the largest anticipated improvement as the optimal level for that factor.

Step 5 Repeat steps 3 and 4 to evaluate the performance of each DMU using model II instead of Model I.

Step 6 Estimate the anticipated improvement in each response due to setting factors at optimal levels. Then, compare the anticipated improvement by the proposed approach for each response with the anticipated improvements gained by adopting other approaches in previous studies. If the response have different quality loss coefficients, then calculate the reduction of quality loss in each response. Otherwise, calculate the S/N ratio for each response using

$$\mathrm{S/N\ ratio} = -10\log_{10}\left(\frac{1}{K}\sum_{k=1}^{K} y_k^2\right) \quad \text{for STB type response} \tag{5}$$

$$\mathrm{S/N\ ratio} = 10\log_{10}\frac{\mu^2}{\sigma^2} \quad \text{for NTB type response} \tag{6}$$

$$\mathrm{S/N\ ratio} = -10\log_{10}\left(\frac{1}{K}\sum_{k=1}^{K}\frac{1}{y_k^2}\right) \quad \text{for LTB type response} \tag{7}$$

where K is the number of response replicates. The μ and σ are response mean and standard deviation, respectively. Obtain the average S/N ratio for each factor level.

The following section provides several examples on the application of the proposed approach.

Application of the proposed approach

Three frequently investigated case studies are selected to illustrate the proposed approach. In the below case studies, the quality loss coefficients for the multiple responses are set equal. Thus, the S/N ratio will be used to estimate the anticipated improvement in each response.

Optimizing polysilicon deposition process

Phadke (1989) conducted the Taguchi method to improve the quality of polysilicon process for three responses; the surface

Table 1 Experimental data of polysilicon process

DMU_j	Control factor[a]								Inputs		Output	Standard efficiency (E_o)
	e	A	B	C	D	E	F	*e*	Quality loss of thickness (x_{1j})	Surface defects (x_{2j})	Deposition rate (y_{1j})	
DMU_1	1	1	1	1	1	1	1	1	0.00030	0.67	14.5	1.00000
DMU_2	1	1	2	2	2	2	2	2	0.00027	36.22	36.6	0.38025
DMU_3	1	1	3	3	3	3	3	3	0.00025	135.78	41.4	0.22037
DMU_4	1	2	1	1	2	2	3	3	0.00006	17.00	36.1	1.00000
DMU_5	1	2	2	2	3	3	1	1	0.00719	1,087.78	73.0	0.02626
DMU_6	1	2	3	3	1	1	2	2	0.00051	839.89	49.5	0.09788
DMU_7	1	3	1	2	1	3	2	3	0.00726	776.33	76.6	0.03359
DMU_8	1	3	2	3	2	1	3	1	0.00520	2,065.33	105.4	0.03032
DMU_9	1	3	3	1	3	2	1	2	0.00087	2,200	115.0	0.13343
DMU_{10}	2	1	1	3	3	2	2	1	0.00206	0.89	24.8	1.00000
DMU_{11}	2	1	2	1	1	3	3	2	0.00013	1.00	20.0	1.00000
DMU_{12}	2	1	3	2	2	1	1	3	0.00016	246.56	39.0	0.25200
DMU_{13}	2	2	1	2	3	1	3	2	0.00062	150.11	53.1	0.16001
DMU_{14}	2	2	2	3	1	2	1	3	0.00005	44.44	45.7	1.00000
DMU_{15}	2	2	3	1	2	3	2	1	0.00018	1,359.44	54.8	0.30722
DMU_{16}	2	3	1	3	2	3	1	2	0.00065	14.33	76.8	0.67157
DMU_{17}	2	3	2	1	3	1	2	3	0.00629	2,201.22	105.3	0.02609
DMU_{18}	2	3	3	2	1	2	3	1	0.01438	3,333.33	91.4	0.01227

[a] *e* Indicates empty column

Table 2 The S/N ratio averages for polysilicon process

Response (dB)	Level[a]	Factor						Optimal factor levels using Taguchi method	Overall average (dB)
		A	B	C	D	E	F		
Thickness	1	**35.12**	31.61	**34.39**	31.68	30.52	27.04	$A_1B_3C_1D_2E_2F_3$	31.52
	2	34.91	30.70	27.86	**34.70**	**32.87**	33.67		
	3	24.52	**32.24**	32.30	28.16	31.16	**33.85**		
Surface defects	1	**−24.23**	**−27.55**	**−39.03**	**−39.20**	−51.53	−45.56	$A_1B_1C_1D_1E_2F_2$	−45.36
	2	−50.11	−47.44	−55.99	−46.85	**−40.54**	**−41.58**		
	3	−61.76	−61.10	−41.07	−50.04	−44.03	−48.95		
Deposition rate	1	28.76	32.03	32.80	32.21	34.06	33.81	$A_3B_3C_2D_3E_3F_3$	34.12
	2	34.13	34.78	**35.29**	34.53	33.99	34.10		
	3	**39.46**	**35.54**	34.25	**35.61**	**34.30**	**34.44**		

[a] Optimal levels using Taguchi method for each response are identified by bold type

Table 3 The optimal DMU weights using benevolent formulation for polysilicon process

DMU_j	Model I				Model II			
	δ	Optimal input weights		Optimal output weight	δ	Optimal input weights		Optimal output weight
		v_{1j}^*	v_{2j}^*	u_{1j}^*		v_{1j}^*	v_{2j}^*	u_{1j}^*
DMU_1	0.0327186	21.6778700	0.0000000	0.0004485	0.0327186	21.6778700	0.0000000	0.0004485
DMU_2	0.0016940	21.6637800	0.0000000	0.0000608	0.0016940	21.6637800	0.0000000	0.0000608
DMU_3	0.0028646	0.0000000	0.0000696	0.0000503	0.0028646	0.0000000	0.0000696	0.0000503
DMU_4	0.0005598	21.5656700	0.0000000	0.0000358	0.0005598	21.5656700	0.0000000	0.0000358
DMU_5	0.0017380	25.4842000	0.0000000	0.0000659	0.0017380	25.4842000	0.0000000	0.0000659
DMU_6	0.0082819	0.0000000	0.0000732	0.0001215	0.0082819	0.0000000	0.0000732	0.0001215
DMU_7	0.0024378	25.5297400	0.0000000	0.0000813	0.0024378	25.5297400	0.0000000	0.0000813
DMU_8	0.0025150	0.0000000	0.0000804	0.0000477	0.0025150	0.0000000	0.0000804	0.0000477
DMU_9	0.0147606	0.0000000	0.0000812	0.0002074	0.0147606	0.0000000	0.0000812	0.0002074
DMU_{10}	0.1956822	22.5377500	0.0000000	0.0018721	0.1956822	22.5377500	0.0000000	0.0018721
DMU_{11}	0.0053359	21.5982700	0.0000000	0.0001404	0.0053359	21.5982700	0.0000000	0.0001404
DMU_{12}	0.0075734	0.0000000	0.0000701	0.0001117	0.0075734	0.0000000	0.0000701	0.0001117
DMU_{13}	0.0007723	21.8292900	0.0000000	0.0000408	0.0007723	21.8292900	0.0000000	0.0000408
DMU_{14}	0.0041721	0.0000000	0.0000691	0.0000672	0.0041721	0.0000000	0.0000691	0.0000672
DMU_{15}	0.0434181	0.0000000	0.0000760	0.0005795	0.0434181	0.0000000	0.0000760	0.0005795
DMU_{16}	0.0045817	21.8436000	0.0000000	0.0001242	0.0045817	21.8436000	0.0000000	0.0001242
DMU_{17}	0.0022387	0.0000000	0.0000812	0.0000443	0.0022387	0.0000000	0.0000812	0.0000443
DMU_{18}	0.0011926	31.2012500	0.0000000	0.0000602	0.0011926	31.2012500	0.0000000	0.0000602

defects (STB), thickness (NTB, target is 3,600 Å) and deposition rate (LTB) are the main responses. Six process factors were investigated simultaneously including: (A) deposition temperature, (B) deposition pressure, (C) nitrogen flow, (D) silane flow, (E) settling time, and (F) cleaning method, utilizing L_{18} $(2^1 \times 3^7)$ array shown in Table 1. The Taguchi method utilizes the S/N ratio to decide the optimal factor levels for each response. Typically, in this method higher S/N ratio indicates better performance. Using the appropriate formula from Eqs. (5) to (7) for each type response, the average S/N ratio for each factor level is calculated and displayed in Table 2.

From Table 2, the optimal factor levels for thickness, surface defects, and deposition rate are identified as $A_1B_3C_1D_2E_2F_3$, $A_1B_1C_1D_1E_2F_2$, and $A_3B_3C_2D_3E_3F_3$, respectively. Obviously, there exists a conflict among these combinations about the optimal factor levels for optimizing the three responses concurrently. To optimize the three responses concurrently by the proposed approach, the following steps were conducted:

Table 4 The cross-efficiencies matrix by benevolent formulation model I and II for polysilicon process

DMU_o	DMU_j																	
	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}	DMU_{11}	DMU_{12}	DMU_{13}	DMU_{14}	DMU_{15}	DMU_{16}	DMU_{17}	DMU_{18}
DMU_1		2.8486	3.4261	12.5140	0.2102	2.0029	0.2184	0.4194	2.7303	0.2485	3.1421	5.1566	1.7693	20.4629	6.2866	2.4366	0.3464	0.1315
DMU_2	0.1356		0.4645	1.6967	0.0285	0.2716	0.0296	0.0569	0.3702	0.0337	0.4260	0.6991	0.2399	2.7744	0.8523	0.3304	0.0470	0.0178
DMU_3	15.6416	0.7303		1.5348	0.0485	0.0426	0.0713	0.0369	0.0378	20.1395	14.4550	0.1143	0.2557	0.7432	0.0291	3.8735	0.0346	0.0198
DMU_4	0.0803	0.2288	0.2752		0.0169	0.1609	0.0175	0.0337	0.2193	0.0200	0.2524	0.4142	0.1421	1.6438	0.5050	0.1957	0.0278	0.0106
DMU_5	0.1249	0.3561	0.4283	1.5644		0.2504	0.0273	0.0524	0.3413	0.0311	0.3928	0.6446	0.2212	2.5581	0.7859	0.3046	0.0433	0.0164
DMU_6	35.9422	1.6782	0.5064	3.5267	0.1115		0.1639	0.0848	0.0868	46.2778	33.2155	0.2627	0.5875	1.7079	0.0669	8.9007	0.0794	0.0455
DMU_7	0.1537	0.4383	0.5272	1.9256	0.0323	0.3082		0.0645	0.4201	0.0382	0.4835	0.7935	0.2723	3.1487	0.9673	0.3749	0.0533	0.0202
DMU_8	12.8579	0.6004	0.1812	1.2616	0.0399	0.0350	0.0586		0.0311	16.5554	11.8825	0.0940	0.2102	0.6110	0.0240	3.1841	0.0284	0.0163
DMU_9	55.2423	2.5794	0.7783	5.4205	0.1713	0.1504	0.2519	0.1303		71.1279	51.0515	0.4038	0.9029	2.6249	0.1029	13.6802	0.1221	0.0700
DMU_{10}	4.0109	11.4366	13.7551	50.2412	0.8438	8.0413	0.8770	1.6839	10.9617		12.6148	20.7025	7.1035	82.1543	25.2393	9.7825	1.3907	0.5281
DMU_{11}	0.3139	0.8949	1.0764	3.9315	0.0660	0.6292	0.0686	0.1318	0.8578	0.0781		1.6200	0.5559	6.4288	1.9750	0.7655	0.1088	0.0413
DMU_{12}	34.4788	1.6099	0.4858	3.3831	0.1069	0.0939	0.1572	0.0813	0.0833	44.3936	31.8631		0.5636	1.6383	0.0642	8.5383	0.0762	0.0437
DMU_{13}	0.0902	0.2572	0.3094	1.1300	0.0190	0.1809	0.0197	0.0379	0.2466	0.0224	0.2837	0.4656		1.8478	0.5677	0.2200	0.0313	0.0119
DMU_{14}	21.0451	0.9826	0.2965	2.0650	0.0653	0.0573	0.0959	0.0496	0.0508	27.0969	19.4486	0.1538	0.3440		0.0392	5.2116	0.0465	0.0267
DMU_{15}	164.9385	7.7013	2.3238	16.1841	0.5115	0.4492	0.7520	0.3889	0.3984	212.3688	152.4260	1.2055	2.6960	7.8374		40.8455	0.3646	0.2090
DMU_{16}	0.2745	0.7826	0.9412	3.4379	0.0577	0.5502	0.0600	0.1152	0.7501	0.0683	0.8632	1.4166	0.4861	5.6216	1.7271		0.0952	0.0361
DMU_{17}	11.8033	0.5511	0.1663	1.1582	0.0366	0.0321	0.0538	0.0278	0.0285	15.1975	10.9078	0.0863	0.1929	0.5609	0.0220	2.9230		0.0150
DMU_{18}	0.0932	0.2658	0.3197	1.1676	0.0196	0.1869	0.0204	0.0391	0.2548	0.0232	0.2932	0.4811	0.1651	1.9093	0.5866	0.2273	0.0323	
e_j	21.0133	1.9966	1.5448	6.5966	0.1403	0.7908	0.1731	0.2020	1.0511	26.6895	20.2354	2.0420	0.9828	8.4867	2.3436	5.9879	0.1722	0.0741
Ordinal value	17	10	9	14	2	6	4	5	8	18	16	11	7	15	12	13	3	1

Table 5 The anticipated improvement for polysilicon process

Response (dB)	Starting condition (1)	Optimal condition (2)					Anticipated improvement (2)–(1)				
		Engineering judgment (Phadke 1989)	Weighted quality loss (Tong et al. 1997)	PCA (Su and Tong 1997)	DEAR (Liao and Chen 2002)	Proposed approach	Engineering judgment (Phadke 1989)	Weighted quality loss (Tong et al. 1997)	PCA (Su and Tong 1997)	DEAR (Liao and Chen 2002)	Proposed approach
Thickness	29.95	36.79	40.24	41.23	41.32	44.79	6.84	10.29	11.28	11.37	14.84
Surface defects	−56.69	−19.84	−24.22	−2.29	1.20	7.03	36.85	32.47	54.40	57.89	63.72
Deposition rate	34.97	29.60	32.44	27.21	27.21	25.64	−5.37	−2.53	−7.76	−7.76	−9.34
Total anticipated improvement (dB)							38.32	40.23	57.92	61.5	69.22

Step 1 The $L_{18}(2^1 \times 3^7)$ array contains 18 experiments. Each experiment is treated as a DMU as shown in the first column of Table 1. The quality loss of thickness, calculated using Eq. (3), and surface defects are set the inputs. However, the deposition rate is set as the output for all DMUs.

Step 2 The standard efficiency, $E_o(o = 1, \ldots, 18)$ is calculated by solving the CCR model for each DMU and also displayed in the last column of Table 1. Note that all the E_o values lie between zero and one, while the E_o value for each of DMU_1, DMU_4, DMU_{10}, DMU_{11}, and DMU_{14} is equal to one. Thus, these DMUs are identified as CCR-efficient. This shows the weakness of the CCR model in discriminating efficient DMUs.

Step 3 Model I is adopted to evaluate the v_{1j}^*, v_{2j}^*, and u_{1j}^* values for each DMU_j. The results are shown in the columns entitled by "Model I" in Table 3.

For illustration, the values of v_{11}^*, v_{21}^*, and u_{11}^* for DMU_1 of 21.6778700, 0.0, and 0.0004485, respectively, are obtained by solving model I as shown below

$$\begin{aligned}
\text{Max} \quad & u_{11}.\sum_{j=2}^{18} y_{1j} - \left(v_{11}.\sum_{j=2}^{18} x_{1j} + v_{21}.\sum_{j=2}^{18} x_{2j} \right) \\
\text{subject to} \quad & \sum_{i=1}^{2} \left(v_{i1}.\sum_{j=2}^{18} x_{ij} \right) = 1 \\
& u_{11} y_{1j} - \sum_{i=1}^{2} v_{i1} x_{ij} \leq \delta \\
& j = 2, \ldots, 18 \\
& u_{11} y_{11} - \sum_{i=1}^{2} v_{i1} x_{i1} = 0 \\
& u_{11}, v_{11}, v_{21} \geq 0
\end{aligned}$$

To avoid infeasible solution, the right hand side of the second constraint is set equal or less than a scalar δ, which is very close to zero as shown in the second column of Table 3. Similarly, the v_{1j}^*, v_{2j}^*, and u_{1j}^* are calculated for the other 17 DMUs. The E_{oj} and e_j values are then calculated for each of the 18 DMUs. Table 4 displays the corresponding cross-efficiencies matrix.

For example, in Table 4 the cross-efficiency, $E_{2,1}$, of DMU_1 evaluated using the optimal weighing scheme of DMU_2 of 0.1356 is calculated as follows. In Table 1, the inputs of DMU_1 are calculated 0.00030 and 0.67, respectively, while the output is estimated 14.5. In Table 3, using model I of benevolent formulation, the calculated v_{12}^*, v_{22}^*, and u_{12}^* for DMU$_2$ are 21.6637800 and 0.0, and 0.0000608, respectively. Substituting these values in Eq. (1) gives

$$\begin{aligned} E_{2,1} &= (0.0000608 \times 14.5)/(21.6637800 \\ &\quad \times 0.0003 + 0.0 \times 0.67) = 0.1356 \end{aligned}$$

In a similar manner, the E_{o1} values of DMU$_1$ are evaluated using the optimal weights of DMU$_3$ to DMU$_{18}$. Using Eq. (2), the mean cross-efficiency of DMU$_1$, e$_1$, is then

$$\mathrm{e}_1 = \sum_{o=2}^{18} E_{o1}/(18 - 1) = 21.0133$$

The e$_j$ values for DMU$_2$ to DMU$_{18}$ are computed similarly. Contrary to the CCR model, the E_{oj} values for some DMUs, as shown in Table 4, are greater than one; for example, the values of $E_{3,1}$ and $E_{6,1}$ are equal to 15.641 and 35.9422, respectively. Moreover, the DMUs identified as CCR-efficient by CCR model have unequal e$_j$ values and thus no more equally efficient using benevolent formulation. This shows that the efficiency of benevolent formulation in increasing the discrimination among efficient DMUs.

Step 4 The ordinal values for all e$_j$ values are listed in the last row of Table 4, where the smallest e$_j$ value takes an ordinal value of one, whereas the largest e$_j$ value has an ordinal value of 18. Utilizing the ordinal values, the AOV$_{fl}$ values are calculated for all factor levels and depicted in Fig. 1. For illustration, the AOV$_{A1}$, the efficiency of level 1 for factor A, is calculated as the average of the ordinal values for DMU$_1$, DMU$_2$, DMU$_3$, DMU$_{10}$, DMU$_{11}$, and DMU$_{12}$, then divided by six; numerically, the AOV$_{A1}$ (=13.5) is obtained from $(17+10+9+18+16+11)/6$. The AOV$_{fl}$ values for the other factor levels are obtained similarly. In Fig. 1, the factor level that maximizes the level efficiency is identified as the optimal level for that factor. Based on this, the A$_1$B$_1$C$_1$D$_2$E$_2$F$_2$ is the combination of factor levels that optimizes the three responses concurrently.

Step 5 Solving model II for each DMU, the v_{1j}^*, v_{2j}^*, and u_{1j}^* are calculated and also listed in the columns entitled "Model II" in Table 3. For illustration, the v_{11}^*, v_{21}^*, and u_{11}^* values for DMU$_1$ are obtained by solving the below model

$$\text{Max } u_{11} \cdot \sum_{j=2}^{18} y_{1j}$$

$$\begin{aligned} \text{subject to} \quad & \sum_{i=1}^{2}\left(v_{i1} \cdot \sum_{j=2}^{18} x_{ij}\right) = 1 \\ & u_{11} y_{1j} - \sum_{i=1}^{2} v_{i1} x_{ij} \le \delta \\ & j = 2, \ldots, 18 \\ & u_{11} y_{11} - \sum_{i=1}^{2} v_{i1} x_{i1} = 0 \\ & u_{11}, v_{11}, v_{21} \ge 0 \end{aligned}$$

It is noted that both models provide the same v_{1j}^*, v_{2j}^*, and u_{1j}^* values for all DMUs. As a result, the cross-efficiency matrix corresponding to model II is similar to that in Table 4. Consequently, the A$_1$B$_1$C$_1$ D$_2$E$_2$F$_2$ is the combination of factor levels for optimizing the three responses concurrently using Model II. At this point, it is concluded that either model I or model II can be used for solving the multi-response problem in the Taguchi method.

Step 6 The effectiveness of proposed approach for optimizing polysilicon process is checked as follows. The average S/N ratio each factor level is calculated for all factor levels and displayed in Table 2. Then, the anticipated improvement in each response due to setting factors at A$_2$B$_1$C$_1$D$_2$E$_2$F$_2$ is calculated and listed in Table 5. In addition, the anticipated improvements gained by other approaches in previous studies, including the sum of the weighted normalized quality losses (Tong et al. 1997), PCA (Su and Tong 1997), and DEAR (Liao and Chen 2002), are also displayed in Table 5.

In Table 5, it is observed that the largest anticipated improvements in thickness (=14.84 dB) and surface defects (= 63.72 dB) correspond to the proposed approach. However, the largest anticipated improvement in deposition rate (= −2.53 dB) corresponds to the sum of the weighted of normalized quality losses. Nevertheless, among all techniques, the proposed approach provides the largest total anticipated improvement (= 69.22 dB). As a result, the proposed approach is superior to engineering judgment, the sum of the weighted normalized quality losses, PCA, and DEAR for solving the multiple responses problem in the Taguchi method for the polysilicon process.

Optimizing gear hobbing operation

Jeyapaul et al. (2006) conducted genetic algorithm to optimize four STB type responses of gear hobbing operation involving: left profile (LP) error, right profile (RP) error, left helix (LH) error, and right helix (RH) error. Six

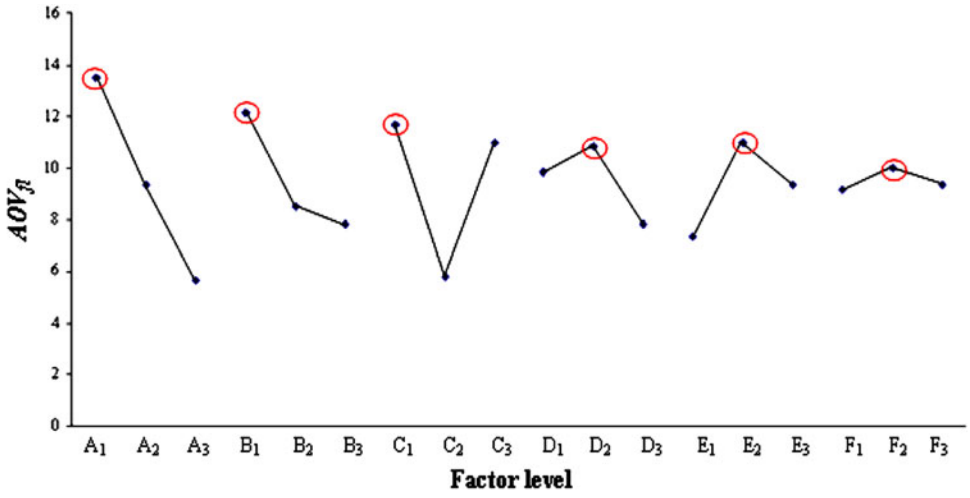

Fig. 1 Optimal factor levels for polycilicon process

Table 6 Experimental data of gear hobbing operation

DMU_j	Control factor								Inputs				Output (y_{1j})	CCR-efficiency (E_o)
	A	BC	D	E	F	Empty			LP error (x_{1j})	RP error (x_{2j})	LH error (x_{3j})	RH error (x_{4j})		
DMU_1	1	1	1	1	1	1	1	1	72.53	73.97	47.37	42.90	1	0.996769
DMU_2	1	1	2	2	2	2	2	2	75.67	74.23	32.43	39.10	1	1.000000
DMU_3	1	1	3	3	3	3	3	3	74.20	73.10	51.93	51.10	1	0.995628
DMU_4	1	2	1	1	2	2	3	3	74.80	77.03	61.27	55.03	1	0.960339
DMU_5	1	2	2	2	3	3	1	1	75.37	75.93	82.97	59.80	1	0.965977
DMU_6	1	2	3	3	1	1	2	2	71.83	73.93	35.83	42.30	1	1.000000
DMU_7	1	3	1	2	1	3	2	3	75.10	71.97	54.47	60.07	1	1.000000
DMU_8	1	3	2	3	2	1	3	1	77.03	74.80	56.17	44.90	1	0.972930
DMU_9	1	3	3	1	3	2	1	2	77.63	72.27	57.87	59.83	1	0.995866
DMU_{10}	2	1	1	3	3	2	2	1	73.67	76.80	42.33	47.10	1	0.975113
DMU_{11}	2	1	2	1	1	3	3	2	74.23	79.03	48.83	34.20	1	0.969096
DMU_{12}	2	1	3	2	2	1	1	3	71.97	75.37	42.03	30.77	1	1.000000
DMU_{13}	2	2	1	2	3	1	3	2	75.10	74.53	34.17	34.73	1	1.000000
DMU_{14}	2	2	2	3	1	2	1	3	76.50	74.50	40.33	37.83	1	0.992851
DMU_{15}	2	2	3	1	2	3	2	1	72.83	74.77	42.33	40.37	1	0.991241
DMU_{16}	2	3	1	3	2	3	1	2	75.63	78.73	45.17	35.27	1	0.952392
DMU_{17}	2	3	2	1	3	1	2	3	75.40	77.07	42.93	39.27	1	0.963499
DMU_{18}	2	3	3	2	1	2	3	1	75.90	72.00	50.90	47.40	1	1.000000

controllable factors were investigated including: (A) direction of hobbing, (B) number of passes, (C) source of hob, (D) feed, (E) speed, and (F) job run out. The $L_{18}(2^1 \times 3^7)$ array was used for providing the layout of experimental work. The proposed approach was implemented for solving multi-response problem for gear hobbing operation and described as follows. Each experiment is treated as a DMU with the LP error, RP error, LH error, and RH, are set as the inputs, whereas one is set as the output for all DMUs as shown in Table 6. By solving the CCR model for each DMU, the E_o values are obtained and also displayed in Table 6. Based on the weighing scheme of the CCR model, the E_o value is equal to one for DMU_2, DMU_6, DMU_7, DMU_{12}, DMU_{13}, and DMU_{18}, and thus these DMUs are considered equally CCR-efficient DMUs. Here also, the CCR model fails to discriminate among these efficient DMUs.

Benevolent formulation models are individually applied to measure performance for each of the 18 DMUs. The optimal input and output weights of each DMU are displayed in Table 7. It is found that both models provide the same weighing scheme for each DMU.

Utilizing the E_o values listed in Table 6 and the optimal input and outputs weights displayed in Table 7, the

Table 7 The optimal weighing scheme by benevolent formulation for gear hobbing operation

DMU_j	δ	Optimal input weights				Optimal input weight
		v^*_{1j}	v^*_{2j}	v^*_{3j}	v^*_{4j}	u^*_{1j}
DMU_1	0.00138003	0.00000000	0.00078366	0.00000000	0.00000000	0.05778020
DMU_2	0.00302429	0.00078758	0.00000000	0.00000000	0.00000000	0.05959582
DMU_3	0.02677772	0.00000000	0.00000000	0.00000000	0.00133179	0.06775685
DMU_4	0.03268316	0.00000000	0.00000000	0.00123753	0.00000000	0.07281632
DMU_5	0.06068102	0.00000000	0.00000000	0.00127168	0.00000000	0.10192170
DMU_6	0.00153593	0.00000000	0.00078364	0.00000000	0.00000000	0.05793433
DMU_7	0.03949319	0.00000000	0.00000000	0.00000000	0.00134789	0.08096779
DMU_8	0.02732483	0.00000000	0.00000000	0.00122977	0.00000000	0.06720628
DMU_9	0.03882377	0.00000000	0.00000000	0.00000000	0.00134746	0.08028495
DMU_{10}	0.02008005	0.00000000	0.00000000	0.00000000	0.00132473	0.06084204
DMU_{11}	0.00363307	0.00000000	0.00078678	0.00000000	0.00000000	0.06025777
DMU_{12}	0.00266738	0.00000000	0.00078452	0.00000000	0.00000000	0.05912949
DMU_{13}	0.00257422	0.00078722	0.00000000	0.00000000	0.00000000	0.05912036
DMU_{14}	0.00324940	0.00078809	0.00000000	0.00000000	0.00000000	0.05985793
DMU_{15}	0.00168210	0.00000000	0.00078415	0.00000000	0.00000000	0.05811765
DMU_{16}	0.00236911	0.00000000	0.00078660	0.00000000	0.00000000	0.05898046
DMU_{17}	0.00179647	0.00000000	0.00078557	0.00000000	0.00000000	0.05833399
DMU_{18}	0.02203904	0.00000000	0.00000000	0.00000000	0.00132526	0.06281723

cross-efficiencies matrix for gear hobbing operation is constructed and shown in Table 8 for both benevolent formulation models.

In order to identify the optimal combination of factor levels, the AOV_{fl} values are calculated and plotted in Fig. 2. In this figure, it is noted that the combination of optimal factor levels can be either $A_2B_1C_1D_2E_2F_2$ or $A_2B_1C_1D_3E_2F_2$, since the optimal level for factor D can be either D_2 or D_3. In other words, a tie occurs in selecting the level of factor D.

To decide whether to adopt $A_2B_1C_1D_2E_2F_2$ or $A_2B_1C_1D_3E_2F_2$, the anticipated improvement in each response is calculated at $A_2B_1C_1D_2E_2F_2$ and $A_2B_1C_1D_3E_2F_2$. The anticipated improvement gained by genetic approach (Jeyapaul et al. 2006) is also displayed in Table 9.

From Table 9, two main conclusions are obtained. First, the total anticipated improvement (= 11.2506 dB), due to setting factor levels at $A_2B_1C_1D_3E_2F_2$, are slightly larger than total anticipated improvement (= 10.6588 dB) due to setting factor levels at $A_2B_1C_1D_2E_2F_2$. As a result, $A_2B_1C_1D_3E_2F_2$ is the optimal combination of factor levels. Secondly, setting factor levels at either combination provides much larger total anticipated improvement than genetic algorithm (= 4.1498 dB). Consequently, the proposed approach is concluded more effective than genetic algorithm for solving multi-response problem in the Taguchi method for the gear hobbing operation.

Optimizing hard disk drive

This case study was performed by the Industrial Technology Research Institute in Taiwan to improve the quality of hard disk drive. The 50% pulse width (PW), peak shift (PS), over write (OW), and high-frequency amplitude (HFA) were the four responses of main interest. The PW and PS are STB type responses, whereas OW and HFA are LTB type responses. Five controllable process factors were investigated, involving: (A) disk writability, (B) magnetization width, (C) gap length, (D) coercivity of media, and (E) rotational speed. The L_{18} $(2^1 \times 3^7)$ array was utilized to investigate these factors simultaneously. It was noticed that the OW has negative values with the desired target zero; however the response should be nonnegative (Phadke 1989). Since maximizing a quality response is equivalent to minimizing the negative of that response, the OW is multiplied by minus one and treated as STB type response. Let OW′ equals $-1 \times$ OW. For this case study, consequently, three STB type responses and one LTB type response will be optimized concurrently as shown in Table 10.

The proposed approach is applied to optimize the four responses of hard disk drive as follows. Each experiment is treated as a DMU with the PW, PS, and OW′ are set as the inputs, whereas the HFA is set as the output for all DMUs. The E_o values are calculated by solving the CCR model for each DMU and also shown in Table 10. It is noted

Table 8 The cross-efficiencies matrix for gear hobbing operation

DMU$_o$	DMU$_j$																	
	DMU$_1$	DMU$_2$	DMU$_3$	DMU$_4$	DMU$_5$	DMU$_6$	DMU$_7$	DMU$_8$	DMU$_9$	DMU$_{10}$	DMU$_{11}$	DMU$_{12}$	DMU$_{13}$	DMU$_{14}$	DMU$_{15}$	DMU$_{16}$	DMU$_{17}$	DMU$_{18}$
DMU$_1$		0.9932	1.0086	0.9571	0.9710	0.9973	1.0245	0.9857	1.0203	0.9600	0.9329	0.9783	0.9892	0.9897	0.9861	0.9365	0.9567	1.0240
DMU$_2$	1.0432		1.0198	1.0116	1.0040	1.0534	1.0076	0.9823	0.9747	1.0272	1.0194	1.0515	1.0076	0.9892	1.0389	1.0005	1.0036	0.9970
DMU$_3$	1.1859	1.3012		0.9245	0.8508	1.2028	0.8470	1.1331	0.8503	1.0802	1.4876	1.6536	1.4648	1.3448	1.2604	1.4426	1.2957	1.0733
DMU$_4$	1.2422	1.8142	1.1330		0.7092	1.6420	1.0803	1.0476	1.0168	1.3899	1.2049	1.3998	1.7221	1.4588	1.3899	1.3027	1.3705	1.1560
DMU$_5$	1.6921	2.4711	1.5433	1.3082		2.2367	1.4715	1.4270	1.3850	1.8932	1.6412	1.9068	2.3458	1.9871	1.8932	1.7745	1.8668	1.5746
DMU$_6$	0.9995	0.9959	1.0114	0.9597	0.9736		1.0273	0.9884	1.0230	0.9626	0.9354	0.9809	0.9919	0.9923	0.9888	0.9390	0.9593	1.0268
DMU$_7$	1.4002	1.5363	1.1755	1.0915	1.0045	1.4201		1.3379	1.0040	1.2754	1.7564	1.9524	1.7295	1.5878	1.4881	1.7033	1.5298	1.2673
DMU$_8$	1.1538	1.6850	1.0523	0.8920	0.6587	1.5251	1.0034		0.9444	1.2909	1.1191	1.3001	1.5995	1.3549	1.2909	1.2100	1.2729	1.0737
DMU$_9$	1.3889	1.5239	1.1660	1.0827	0.9964	1.4086	0.9919	1.3270		1.2650	1.7422	1.9366	1.7154	1.5749	1.4760	1.6895	1.5174	1.2570
DMU$_{10}$	1.0706	1.1746	0.8988	0.8345	0.7680	1.0858	0.7646	1.0229	0.7676		1.3429	1.4928	1.3223	1.2140	1.1378	1.3023	1.1696	0.9689
DMU$_{11}$	1.0354	1.0317	1.0477	0.9942	1.0086	1.0359	1.0642	1.0239	1.0598	0.9972		1.0162	1.0276	1.0280	1.0244	0.9727	0.9938	1.0637
DMU$_{12}$	1.0190	1.0153	1.0311	0.9784	0.9926	1.0194	1.0473	1.0076	1.0429	0.9814	0.9536		1.0112	1.0117	1.0081	0.9573	0.9780	1.0468
DMU$_{13}$	1.0354	0.9925	1.0121	1.0040	0.9965	1.0455	1.0000	0.9749	0.9674	1.0195	1.0117	1.0435		0.9817	1.0311	0.9929	0.9960	0.9895
DMU$_{14}$	1.0471	1.0038	1.0236	1.0154	1.0078	1.0574	1.0114	0.9860	0.9784	1.0310	1.0232	1.0554	1.0114		1.0428	1.0042	1.0073	1.0007
DMU$_{15}$	1.0020	0.9984	1.0139	0.9621	0.9761	1.0025	1.0299	0.9908	1.0256	0.9650	0.9378	0.9834	0.9944	0.9948		0.9413	0.9617	1.0294
DMU$_{16}$	1.0137	1.0101	1.0257	0.9734	0.9875	1.0142	1.0419	1.0024	1.0376	0.9763	0.9487	0.9949	1.0060	1.0065	1.0029		0.9729	1.0414
DMU$_{17}$	1.0039	1.0003	1.0158	0.9640	0.9779	1.0044	1.0318	0.9927	1.0275	0.9669	0.9396	0.9853	0.9963	0.9967	0.9932	0.9431		1.0313
DMU$_{18}$	1.1049	1.2123	0.9276	0.8613	0.7926	1.1206	0.7891	1.0557	0.7922	1.0064	1.3860	1.5406	1.3647	1.2529	1.1742	1.3440	1.2071	
e_j	1.1434	2.4178	2.0118	1.8683	1.7417	2.3190	1.9148	2.0318	1.8797	2.1209	2.2647	2.4747	2.4777	2.3073	2.2474	2.2730	2.2288	2.0691
Ordinal value	1	16	6	3	2	15	5	7	4	9	12	17	18	14	11	13	10	8

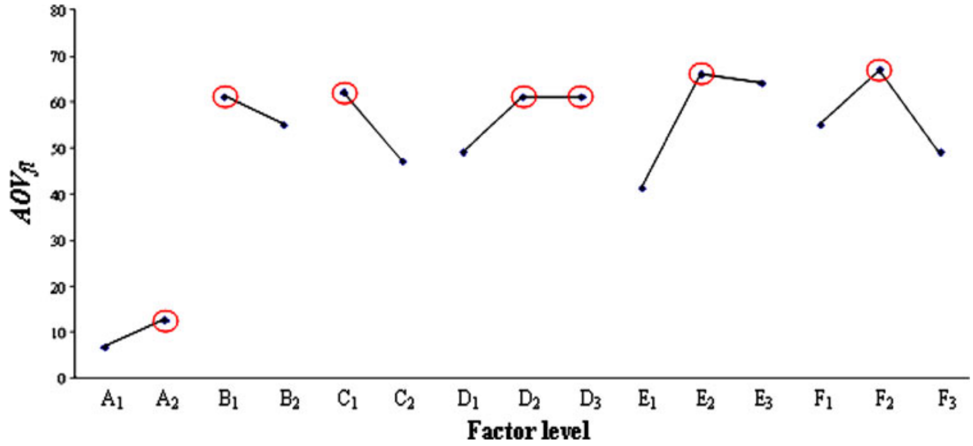

Fig. 2 Optimal factor levels for gear hobbing operation

Table 9 Summary of anticipated improvement for gear hobbing operation

Response (dB)	Initial condition (1)	Optimal condition (2)			Anticipated improvement (2)–(1)		
		Genetic algorithm (Jeyapaul et al. 2006)	Proposed approach		Genetic algorithm (Jeyapaul et al. 2006)	Proposed approach	
			$A_2B_1C_1D_2E_2F_2$	$A_2B_1C_1D_3E_2F_2$		$A_2B_1C_1D_2E_2F_2$	$A_2B_1C_1D_3E_2F_2$
LP error	−37.8581	−37.4917	−37.3728	−37.1800	0.3664	0.4853	0.6781
RP error	−37.4952	−37.4045	−37.7724	−37.4984	0.0907	−0.2772	−0.0032
LH error	−36.6009	−34.4082	−31.9040	−31.4320	2.1927	4.6968	5.1688
RH error	−35.7397	−34.2396	−29.9858	−30.3328	1.5001	5.7539	5.4069
Total anticipated improvement (dB)					4.1498	10.6588	11.2506

Table 10 Experimental data of hard disk drive

DMU_j	Control factors								Input			Output	CCR-efficiency (E_o)
	A	B	C	D	E	Empty			PW (x_{1j})	PS (x_{2j})	OW′ (x_{3j})	HFA (y_{1j})	
DMU_1	1	1	1	1	1	1	1	1	64.75	11.45	31.15	272.15	0.60996
DMU_2	1	1	2	2	2	2	2	2	65.10	12.30	34.05	326.80	0.68183
DMU_3	1	1	3	3	3	3	3	3	66.30	14.15	35.75	367.75	0.66695
DMU_4	1	2	1	1	2	2	3	3	55.55	10.00	32.50	311.75	0.80002
DMU_5	1	2	2	2	3	3	1	1	57.00	10.70	35.55	350.65	0.84098
DMU_6	1	2	3	3	1	1	2	2	88.40	18.45	39.20	223.90	0.31422
DMU_7	1	3	1	2	1	3	2	3	64.85	10.95	30.60	273.60	0.64121
DMU_8	1	3	2	3	2	1	3	1	65.20	11.40	34.55	320.35	0.72113
DMU_9	1	3	3	1	3	2	1	2	66.25	14.90	45.10	297.75	0.51859
DMU_{10}	2	1	1	3	3	2	2	1	48.60	11.40	18.95	422.40	1.00000
DMU_{11}	2	1	2	1	1	3	3	2	75.95	17.10	33.10	277.30	0.42697
DMU_{12}	2	1	3	2	2	1	1	3	75.70	17.75	34.45	329.60	0.50097
DMU_{13}	2	2	1	2	3	1	3	2	48.60	10.80	24.05	420.85	1.00000
DMU_{14}	2	2	2	3	1	2	1	3	76.00	15.55	29.30	296.65	0.50461
DMU_{15}	2	2	3	1	2	3	2	1	75.70	18.60	38.65	258.65	0.39312
DMU_{16}	2	3	1	3	2	3	1	2	55.55	12.50	18.80	360.95	0.86134
DMU_{17}	2	3	2	1	3	1	2	3	57.00	12.75	35.10	360.10	0.72924
DMU_{18}	2	3	3	2	1	2	3	1	88.35	20.35	37.75	257.60	0.33589

Table 11 The optimal weighing scheme by benevolent formulation for gear hobbing operation

DMU_j	δ	Optimal input weights			Optimal output weights
		v^*_{1j}	v^*_{2j}	v^*_{3j}	u^*_{4j}
DMU_1	0.01123747	0.00088488	0.00000000	0.00000000	0.00012842
DMU_2	0.01994025	0.00000000	0.00000000	0.00180326	0.00012811
DMU_3	0.01526047	0.00000000	0.00000000	0.00180881	0.00011728
DMU_4	0.02927371	0.00000000	0.00000000	0.00179824	0.00014998
DMU_5	0.03085495	0.00000000	0.00000000	0.00180816	0.00015417
DMU_6	0.00343709	0.00090379	0.00000000	0.00000000	0.00011212
DMU_7	0.01380309	0.00088496	0.00000000	0.00000000	0.00013450
DMU_8	0.02509146	0.00000000	0.00000000	0.00180489	0.00014037
DMU_9	0.02618165	0.00000000	0.00000000	0.00183993	0.00014453
DMU_{10}	0.00232861	0.00000000	0.00417188	0.00000000	0.00011259
DMU_{11}	0.00119998	0.00000000	0.00427350	0.00000000	0.00011252
DMU_{12}	0.00237426	0.00000000	0.00428541	0.00000000	0.00011562
DMU_{13}	0.00000000	0.00000000	0.00416147	0.00000000	0.00010679
DMU_{14}	0.00536898	0.00089377	0.00000000	0.00000000	0.00011555
DMU_{15}	0.00472010	0.00000000	0.00430108	0.00000000	0.00012159
DMU_{16}	0.00734913	0.00000000	0.00419112	0.00000000	0.00012502
DMU_{17}	0.02000846	0.00000000	0.00000000	0.00180669	0.00012842
DMU_{18}	0.00159118	0.00000000	0.00433369	0.00000000	0.00011499

that DMU_{10} and DMU_{13} are the only CCR-efficient DMUs. Utilizing the E_o values, benevolent formulation models are solved separately to estimate the optimal input and output weights of each of the 18 DMUs as shown in Table 11 for both models.

Since both models provide the same weighing scheme for each DMU, the cross-efficiencies matrix for hard disk drive shown in Table 12 corresponds to both models. As a result, both models provide the same combination of optimal factor levels for multi-responses.

Utilizing the ordinal values in Table 12, the AOV_{fl} values are calculated for each factor level and depicted in Fig. 3. Obviously, the combination of optimal factor levels is identified as $A_2B_1C_1D_3E_3$. The anticipated improvement in each response due to setting factor levels at $A_2B_1C_1D_3E_3$ is calculated and compared with the anticipated improvement using PCA (Su and Tong 1997) and DEAR (Liao and Chen 2002) in Table 13.

In Table 13, the total anticipated improvement gained by the proposed approach is 10.681 dB. Whereas, the total anticipated improvement using PCA and DEAR are 2.61 and 3.35 dB, respectively. Obviously, the proposed approach outperforms PCA and DEAR in solving the multi-response problem in the Taguchi method for hard disk drive.

Conclusions

This research proposes an approach for solving the multi-response problem in the Taguchi method utilizing benevolent formulation in DEA. Three case studies are provided for illustration. From the computational results of three case studies, the following advantages of the proposed approach are noted regarding:

- *Efficiency*: The proposed approach is found the most efficient in solving the multi-response problem in the Taguchi method, as it provides the largest anticipated improvement for all the three cases.
- *Priori information*: In contrast to the Taguchi method and weighted S/N ratios method, the proposed approach does not require any priori information about response weights or importance.
- *Discrimination*: Opposite to CCR-model, the benevolent technique increases discrimination among efficient DMUs.
- *Assumption*: The proposed approach is not based on rigid assumptions, whereas PCA does.
- *Simplicity*: Contrary to GA, neural networks, grey analysis, and regression method, the proposed approach can be easily understood and implemented by practitioners.

Table 12 The cross-efficiencies matrix for hard disk drive

DMU_o	DMU_j																	
	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8	DMU_9	DMU_{10}	DMU_{11}	DMU_{12}	DMU_{13}	DMU_{14}	DMU_{15}	DMU_{16}	DMU_{17}	DMU_{18}
DMU_1		0.72851	0.80496	0.81443	0.89275	0.36757	0.61226	0.71303	0.65223	1.26131	0.52985	0.63186	1.25668	0.56645	0.49585	0.94297	0.91681	0.42313
DMU_2	0.62067		0.73078	0.68145	0.70072	0.40577	0.63519	0.6587	0.46901	1.58353	0.59516	0.67969	1.24315	0.71926	0.47542	1.36396	0.72883	0.48477
DMU_3	0.56646	0.62227		0.62193	0.63951	0.37033	0.57971	0.60116	0.42805	1.44521	0.54317	0.62032	1.13456	0.65644	0.43389	1.24482	0.66517	0.44243
DMU_4	0.72867	0.80047	0.85794		0.82264	0.47637	0.74571	0.77331	0.55062	1.85906	0.69871	0.79795	1.45945	0.84441	0.55814	1.60128	0.85565	0.56912
DMU_5	0.74491	0.81831	0.87706	0.81785		0.48699	0.76234	0.79055	0.56289	1.90049	0.71429	0.81574	1.49198	0.86323	0.57058	1.63697	0.87472	0.58181
DMU_6	0.52144	0.62278	0.68813	0.69623	0.76319		0.52341	0.60955	0.55757	1.07825	0.45295	0.54016	1.07429	0.48424	0.42389	0.80611	0.78375	0.36172
DMU_7	0.6388	0.76295	0.84301	0.85294	0.93496	0.38494		0.74674	0.68306	1.32094	0.5549	0.66174	1.31609	0.59323	0.51929	0.98755	0.96016	0.44313
DMU_8	0.6795	0.74645	0.80004	0.74604	0.76713	0.44423	0.6954		0.51347	1.73361	0.65157	0.74411	1.36097	0.78743	0.52048	1.49323	0.79791	0.53072
DMU_9	0.68628	0.7539	0.80803	0.75348	0.77479	0.44866	0.70233	0.72833		1.75091	0.65807	0.75153	1.37455	0.79529	0.52567	1.50813	0.80587	0.53602
DMU_{10}	0.64148	0.71706	0.70142	0.84137	0.88445	0.32752	0.67435	0.7584	0.53932		0.43766	0.50115	1.05168	0.51487	0.3753	0.77932	0.76224	0.34164
DMU_{11}	0.62582	0.69955	0.68429	0.82082	0.86285	0.31952	0.65788	0.73988	0.52615	0.97558		0.48891	1.026	0.50229	0.36614	0.76029	0.74363	0.33329
DMU_{12}	0.64125	0.7168	0.70116	0.84106	0.88412	0.3274	0.6741	0.75813	0.53912	0.99964	0.4375		1.0513	0.51468	0.37516	0.77904	0.76197	0.34151
DMU_{13}	0.60996	0.68183	0.66695	0.80002	0.84098	0.31143	0.64121	0.72113	0.51282	0.95086	0.41615	0.47652		0.48957	0.35686	0.74103	0.72479	0.32485
DMU_{14}	0.54337	0.64897	0.71707	0.72552	0.79529	0.32744	0.54542	0.63519	0.58102	1.1236	0.47201	0.56288	1.11948		0.44171	0.84002	0.81672	0.37693
DMU_{15}	0.67194	0.75111	0.73472	0.88132	0.92644	0.34307	0.70636	0.79441	0.56493	1.04748	0.45844	0.52495	1.10161	0.53931		0.81632	0.79843	0.35785
DMU_{16}	0.70899	0.79253	0.77524	0.92992	0.97752	0.36199	0.74531	0.83822	0.59608	1.10524	0.48372	0.55389	1.16236	0.56905	0.4148		0.84246	0.37759
DMU_{17}	0.62102	0.68221	0.73119	0.68183	0.70111	0.406	0.63555	0.65907	0.46928	1.58442	0.59549	0.68007	1.24385	0.71967	0.47568	1.36472		0.48505
DMU_{18}	0.63069	0.70501	0.68962	0.82722	0.86957	0.32201	0.66301	0.74565	0.53025	0.98318	0.4303	0.49273	1.034	0.50621	0.36899	0.76622	0.74943	
e_j	0.64007	0.72063	0.75362	0.78432	0.82577	0.37831	0.6588	0.72185	0.54564	1.33549	0.53705	0.61907	1.206	0.62739	0.45281	1.08423	0.79933	0.43009
Ordinal value	8	10	12	13	15	1	9	11	5	18	4	6	17	7	3	16	14	2

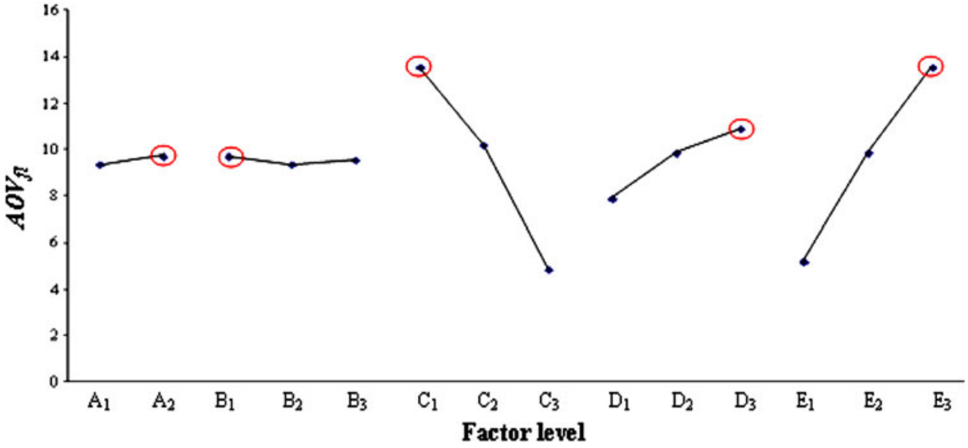

Fig. 3 Optimal factor levels for hard disk drive

Table 13 Summary of anticipated improvement for hard disk drive

Response (dB)	Initial condition (1)	Optimal condition (2)			Anticipated improvement (2)–(1)		
		PCA (Su and Tong 1997)	DEAR (Liao and Chen 2002)	Proposed approach	PCA (Su and Tong 1997)	DEAR (Liao and Chen 2002)	Proposed approach
PW	−36.28	−33.74	−33.74	−33.734	2.54	2.54	2.543
PS	−21.48	−19.37	−19.17	−21.045	2.11	2.31	0.435
OW	31.51[a]	27.71	28.97	−25.669	−3.80	−2.54	5.219
HFA	50.47	52.23	51.51	52.949	1.76	1.04	2.484
Total anticipated improvement (dB)					2.61	3.35	10.681

[a] Initial condition is multiplied by minus one, and thus initial condition is −31.51

The above advantages may make the proposed approach be used for solving the multi-response problem for a wide range of applications in manufacturing on the Taguchi method. In conclusion, benevolent formulation is not only efficient in comparison among DMUs at organizational level, but also effective for solving the multi-response problem in manufacturing at operational level. Future research will be conducted to solve the multi-response problem in the Taguchi method with correlated multiple responses in the Taguchi method utilizing DEA techniques.

Appendix A: CCR-model

Assuming there are n DMUs each with m inputs and s outputs to be evaluated. Let the DMU to be individually evaluated on any trial be designated as DMU$_o$, where o ranges from one to n. The relative efficiency, E_o, of DMU$_o$ with inputs of x_{io} $(i = 1, \ldots, m)$ and outputs of y_{ro} $(r = 1, \ldots, s)$ is evaluated by CCR model as follows:

$$E_o = \operatorname{Max}\theta = \left(\sum_{r=1}^{s} u_r y_{ro}\right) \Big/ \left(\sum_{i=1}^{m} v_i x_{io}\right)$$

$$\begin{aligned}\text{subject to}\quad & \left(\sum_{r=1}^{s} u_r y_{rj}\right) \Big/ \left(\sum_{i=1}^{m} v_i x_{ij}\right) \le 1\\ & j = 1, \ldots, n\\ & u_1, u_2, \ldots, u_s \ge 0\\ & v_1, v_2, \ldots, v_m \ge 0\end{aligned}$$

where u_r and v_i are the virtual weights for the rth output and ith input, respectively, and θ is a scalar. The objective function is the ratio of the sum of the weighted outputs relative to the sum of the weighted inputs. The first constraint ensures that E_j lies between zero and one for all the n DMUs. Obviously, the CCR model is nonlinear, which can be transformed into a linear model by setting the sum of the weighted inputs equal to one. The resulting model is called the "input-oriented" CCR model, which is expressed as follows:

$$\begin{aligned}E_o = \operatorname{Max}\theta = & \sum_{r=1}^{s} u_r y_{ro}\\ \text{subject to}\quad & \sum_{i=1}^{m} v_i x_{io} = 1\\ & \sum_{r=1}^{s} u_r y_{rj} \le \sum_{i=1}^{m} v_i x_{ij} \quad j = 1, \ldots, n\end{aligned}$$

$$
\begin{aligned}
& u_1, u_2, \ldots, u_s \geq 0 \\
& v_1, v_2, \ldots, v_m \geq 0
\end{aligned}
$$

Appendix B: Benevolent formulation models

Model I

This model is expressed as:

$$
\begin{aligned}
\text{Max} \quad & \sum_{r=1}^{s}\left(u_{ro}.\sum_{j\neq o} y_{rj}\right) - \sum_{i=1}^{m}\left(v_{io}.\sum_{j\neq o} x_{ij}\right) \\
\text{subject to} \quad & \sum_{i=1}^{m}\left(v_{io}.\sum_{j\neq o} x_{ij}\right) = 1, \\
& \sum_{r=1}^{s} u_{ro}y_{rj} - \sum_{i=1}^{m} v_{io}x_{ij} \leq 0, \forall j \neq o \\
& \sum_{r=1}^{s} u_{ro}y_{ro} - E_o.\sum_{i=1}^{m} v_{io}x_{io} = 0, \\
& u_{ro}, v_{io} \geq 0, \quad \forall r, \forall i
\end{aligned}
$$

In the this model, the decision variables are u_{ro} and v_{io}. For DMU$_o$, the objective function would presumably seek to maximize the other DMU's cross-efficiencies when measuring them by applying its own best weights. The first constraints is used to transform the objective function into linear function by setting the sum of other $(n - 1)$ DMUs inputs and outputs weighted by DMU$_o$'s input and output weights, respectively, equal to one. The second constraint guarantees that the relative efficiency scores for all DMUs; expect DMU$_o$, are less than one. The third constraint keeps the relative efficiency of DMU$_o$ calculated by CCR-Model equal to E_o. The last constraint is to obtain positive optimal values for the decision variables v_{io} and u_{ro}.

Model II

In Model II, the objective function is to maximize the sum of the weighted outputs, while the constraints are similar to Model I. Mathematically,

$$
\begin{aligned}
\text{Max} \quad & \sum_{r=1}^{s}\left(u_{ro}.\sum_{j\neq o} y_{rj}\right) \\
\text{subject to} \quad & \sum_{i=1}^{m}\left(v_{io}.\sum_{j\neq o} x_{ij}\right) = 1 \\
& \sum_{r=1}^{s} u_{ro}y_{rj} - \sum_{i=1}^{m} v_{io}x_{ij} \leq 0, \quad \forall j \neq o \\
& \sum_{r=1}^{s} u_{ro}y_{ro} - E_o.\sum_{i=1}^{m} v_{io}x_{io} = 0 \\
& u_{ro}, v_{io} \geq 0
\end{aligned}
$$

Similar to Model I, the decision variables are u_{ro} and v_{io}.

References

Al-Refaie, A., Li, M. H. C., & Tai, K. C. (2008). Optimizing SUS 304 wire drawing process by grey analysis utilizing Taguchi method. *Journal of University of Science and Technology Beijing, 15*(6), 714–722.

Anastasiou, K. S. (2002). Optimization of the aluminium die casting process based on the Taguchi method. *Proceedings of the Institution of Mechanical Engineers, 216*(7), 969–977.

Angulo-Meza, L., & Lins, M. P. E. (2002). Review of methods for increasing discrimination in data envelopment analysis. *Annuals of Operations Research, 116*, 225–242.

Antony, J. (2000). Multi-response optimization in industrial experiments using Taguchi's quality loss function and principal component analysis. *Quality and Reliability Engineering International, 16*, 3–8.

Baker, R. C., & Talluri, S. (1997). A closer look at the use of data envelopment analysis for technology selection. *Computers and Industrial Engineering, 28*, 101–108.

Charnes, A., Cooper, W. W., Huang, Z. M., & Sun, D. B. (1990). Polyhedral cone-ratio DEA models with an illustrative application to large commercial banks. *Journal of Econometrics, 46*, 73–91.

Charnes, A., Cooper, W. W., Letwin, A. A., & Seiford, L. M. (1994). *Data envelopment analysis*. Dordrecht: Kluwer.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*, 429–444.

Dyson, R. G., & Thanassoulis, E. (1988). Reducing weight flexibility in data envelopment analysis. *Journal of the Operational Research Society, 39*, 563–576.

Halme, M., Joro, T., Korhonen, P., Salo, S., & Wallenius, J. (2000). Value efficiency analysis for incorporating preference information in DEA. *Management Science, 45*, 103–115.

Jeyapaul, R., Shahabudeen, P., & Krishnaiah, K. (2006). Simultaneous optimization of multi-response problems in the Taguchi method using genetic algorithm. *International Journal of Advanced Manufacturing Technology, 30*, 870–878.

Khouja, M. (1995). The use of data envelopment analysis for technology selection. *Computers and Industrial Engineering, 28*, 123–132.

Li, M. H, Al-Refaie, A., & Yang, C. Y. (2008). DMAIC approach to improve the capability of SMT solder printing process. *IEEE Transactions on Electronics Packaging Manufacturing, 24*, 351–360.

Liao, H. C. (2005). Using N-D method to solve multi-response problem in Taguchi. *Journal of Intelligent Manufacturing, 16*, 331–347.

Liao, H. C., & Chen, Y. K. (2002). Optimizing multi-response problem in the Taguchi method by DEA based ranking method. *International Journal of Quality and Reliability Management, 19*((7), 825–837.

Phadke, M. S. (1989). *Quality engineering using robust design*. Englewood Cliffs, NJ: Prentice-Hall.

Reddy, P. B. S., Nishina, K., & Subash Babu, A. (1997). Unification of robust design and goal programming for multiresponse optimization—a case study. *Quality and Reliability Engineering International, 13*, 371–383.

Sexton, T. R., Slinkman, R. H., & Hogan, A. (1986). Data envelopment analysis: Critique and extensions. In R. H. Slinkman (Ed.), *Measuring efficiency: An assessment of data envelopment analysis* (vol. 32). New directions of program evaluation, Jossey Bass, San Francisco.

Shiau, G. H. (1990). A study of the sintering properties of iron ores using the Taguchi's parameter design. *Journal of the Chinese Statistical Association, 28*, 253–275.

Su, C. T., & Tong, L. I. (1997). Multi-response robust design by principal component analysis. *Total Quality Management, 8*(6), 409–416.

Taguchi, G. (1991). *Taguchi methods. Research and development* (Vol. 1). Dearborn, MI: American Suppliers Institute Press.

Tong, L. I., Su, C. T., & Wang, C. H. (1997). The optimization of multi-response problems in the Taguchi method. *International Journal of Quality and Reliability Management, 14*(4), 367–380.

Tsao, C. C., & Hocheng, H. (2002). Comparison of the tool life of tungsten carbides coated by multi-layer TiCN and TiAlCN for end mills using the Taguchi method. *Journal of Material Processing Technology, 123*, 1–4.

Yu, J. C., Chen, X. X., Hung, T. R., & Thibault, F. (2004). Optimization of extrusion blow molding processes using soft computing and Taguchi's method. *Journal of Intelligent Manufacturing, 15*, 625–634.