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Introduction

Since the introduction of Harris's EOQ model in 1915, many academic researchers and practising managers have become intrigued with developing and implementing effective inventory management systems. Hundreds of articles, such as those surveyed by Aggarwal[1] and Silver[2], attest to the perceived importance of inventory management topics.

A survey of the literature reveals that there have been many refinements to simple reorder point systems and periodic review systems. Starting with the basic EOQ model, many researchers have developed advanced models that reflect more realistic operating environments. These advanced models account for different demand patterns, quantity discounts, stockout costs, lead time variations, and multi-stage, multi-item situations[3]. Efforts to reflect different operating environments resulted in the more recent development of control systems such as MRP, DRP, and JIT[4]. However, the overriding concerns of past authors have been on developing efficient techniques and designing effective control systems.

Since inventory systems function at the direction of human operators within dynamic environments, it is reasonable to assume that a system's effectiveness is subject both to the discipline of the human operators and to changing environmental conditions. In order to ensure that an inventory management system is performing as originally designed, managers must monitor and evaluate its performance on an ongoing basis and, if necessary, take corrective action. Unfortunately, prior research has suggested few concepts or techniques for diagnosing the performance of management systems. Eilon and Elmaleh[5] suggested using a model that would adjust the decision rules when the external environment changed. Pope and Ardalan[6] identified a number of environmental and performance variables that could be monitored but did not develop an integrated monitoring system. However, most of the inventory management literature focuses on prescribing "correct" inventory systems.

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Therefore, the purpose of this article is to begin filling this void. Specifically, we propose the control chart method for monitoring a system's performance on an ongoing basis.

The discussions in this article are confined to a reorder point system for independent demand items. It is hoped that the concepts presented can be extended to other inventory systems which are not so narrowly defined. The remainder of this article identifies sources of performance deviation in reorder point systems, suggests methods for monitoring system performance, and presents a sample problem.

Sources of Performance Deviation

When an inventory system does not perform according to plan, there must be reasons for the performance deviations. Before any corrective actions can be taken the sources of deviations must be identified. These can be broadly classified into two categories:

(1) causes related to system fitness; and

(2) causes related to ongoing operations.

Systems Fitness Causes

One of the most frequent sources of performance deviation is faulty design or selection of a system. Generally, the inventory management environment is very complex involving many different variables, decisions, and situational characteristics. Therefore, in designing or selecting a system, managers must make sure that the chosen system is consistent with the operating environment of the items to be managed. It must be compatible in terms of product characteristics, demand patterns, cost characteristics, and management policies. For example, reorder point and periodic review systems work best when the item's demand pattern is characterized by relatively constant usage. Thus, if the demand pattern of an item appears to be "lumpy", a reorder point system designed with the assumption of relatively constant item usage may not perform well.

Another source of performance deviation is the dynamic nature of the operating environment within which the system functions. A system that was designed or selected for one set of operating conditions could quickly become obsolete as operating conditions change. For example, an item's demand rate, demand pattern, lead time duration, and lead time variability could change. When such changes occur, the system's decision rules must be updated to reflect correctly the relevant environment and, thereby, maintain the desired performance level. In extreme cases, changing operating conditions may necessitate designing an entirely different inventory system. In order to minimize the occurrence and impact of performance deviations caused by system fitness problems, the manager must ensure that the inventory management system is consistent with the operating environment and that the system is monitored to ensure ongoing compatibility.

Operations-related Causes

Once a system has been selected and installed, it must be used according to its design specifications. One of the most frequent causes of system malfunctions is the lack of discipline of the system's operators. They must be disciplined in their inventory withdrawal and recording practices. Failure to maintain this discipline results in inventory record inaccuracies. Since record inaccuracies can be fatal to any inventory management system, the importance of maintaining high levels of accuracy cannot be over-emphasized.

The operating performance of an inventory management system can also deteriorate because of the inability of a *disciplined* operator to execute the system correctly. For example, an operator may be inadvertently assigned responsibility for more product items than can be reasonably handled by one person. In this case, the system's requests to place new orders may be missed, delayed, or acted on more than once. Any of these problems will cause the inventory system to malfunction.

When an inventory system is not functioning properly, the manager must first be alerted to the problem and second be able to pinpoint the cause(s) of the malfunction in timely manner. The remainder of the article describes an approach for monitoring a reorder point inventory system.

Tracking System Performance

A typical inventory management system can be characterized by many environmental variables, decision rules, and performance measures. A variety of variables, rules and measures that fall into each of those categories are listed below:

(1) *Environmental variables*. Variables used to determine appropriate decision rules such as:

- Demand – mean and variance of demand per period.
- Cost – unit cost of the item, inventory- carrying costs, set-up or ordering costs, and stockout costs.
- Lead time – mean and variance of lead time per order.
- Management policies – specific item priorities (e.g. using an ABC classification) or special customer service requirements.

(2) *Decision rules*. Rules used to control the execution of an inventory management system such as:

- Order quantity – the number of units ordered when a replenishment order is required.
- Reorder point – the stock position at which a replenishment order should be placed.
- Safety stock level – the amount of inventory used to protect against uncertainty during replenishment lead time.

(3) *Performance measures*. Measures used to monitor the performance of the inventory management system such as:
- turnover rate – the number of times that the average inventory level could be sold per year if consumed at the average annual rate.
- Stockout frequency and quantity – the number of missed customer orders and their size.
- Inventory to sales ratio – the ratio of average inventory level divided by average sales rate (the inverse of Inventory Turnover Rate).
- Total inventory investment – the total amount of money invested in all types of inventory (RM, WIP and FG).
- Customer order fill rate – the percentage of customer orders filled directly from inventory.

The environmental variables are typically used to design and install the system, which then operates using the appropriate decision rules to signal any necessary actions to the system operator. Finally, the performance measures must be monitored to ensure that the system is functioning properly. Since the system may not be capable of detecting and reacting to changes in its operating environment, a carefully designed monitoring system is required. In this article, we propose using control charts as monitoring devices

A monitoring system can be designed from two distinctive perspectives. The first perspective is to focus on performance deviations attributable to system fitness causes and thus monitor whether any significant changes occur to the environmental variables. If these have changed, the inventory system's performance measures would show corresponding changes. The monitoring system will provide information as to the fitness of the inventory system and also will provide valuable information about the corrective action(s) necessary for its decision rules.

The second perspective would be to focus on performance deviations attributable to operations-related causes, and thus monitor the performance measures directly. In some instances, the inventory system's performance measures can deviate from the norms even if the environmental variables have not changed significantly. When this occurs, the causes of an inventory system's malfunction or deviation can usually be attributed to operations-related causes.

The suggested perspective for designing a monitoring system is to observe the combined effects of both causes. Therefore it is suggested that the best type of monitoring systems tracks both the performance measures and the environmental variables.

Diagnosing System Deviations

The model proposed uses three factors to monitor inventory system performance: two performance measures (inventory turnover rate and

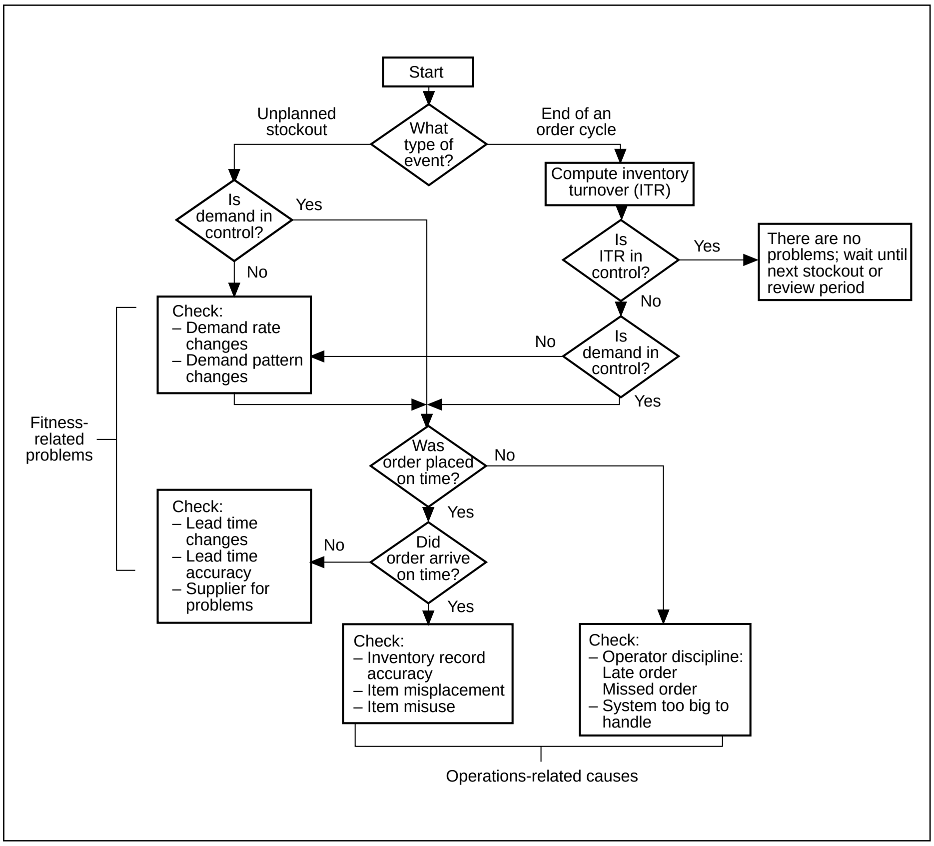

Figure 1.
Inventory-monitoring
System Flowchart

stockouts) and one environmental variable (demand). Figure 1 contains a flowchart of the proposed monitoring system.

Triggered by an unplanned *stockout* or by a periodic review of the *inventory turnover rate* for any items that have just finished an order cycle, the monitoring system is designed to analyse systematically the causes of inventory system deviations. The analysis eventually leads to the fitness-related and/or operations-related causes of the deviations.

Unplanned Stockout

When an unplanned stockout occurs, the inventory manager should be notified and a control chart for demand should be consulted to determine whether demand is in control or not. A technique for establishing the control limits for demand is shown in the Appendix. If the demand during the reorder cycle is outside the control limits, further investigation should be performed to determine if the demand rate or pattern has changed.

After checking for changes in the demand rate or pattern, the timeliness of order placement and arrival should be also checked to ensure that all possible causes for the stockout are identified. If the most recent order was not placed on time, the probable causes for the error are either that operator discipline has been lax (late order or missed order) or that the system is too big for the current staff to handle. Note that both of these are operations-related problems. If the order was placed on time, then the manager should check to see if it arrived on time. If it did, then the inventory data may be inaccurate, the order may have been misplaced, or the item may have been misused. Once again, these are all operations-related problems. If the order did not arrive on time, then a further investigation would likely reveal that the lead time has changed or that a supplier-related problem exists. Both of these are fitness-related problems. It should be noted that a combination of fitness and operations related problems can cause the performance deviation.

End of Order Cycle

In addition to investigating unplanned stockouts, the inventory turnover rate (ITR) should be checked at the end of each order cycle to see if it is out of control. Whenever any item receives a replenishment order its ITR is calculated and examined. A technique for establishing control limits for ITR is also shown in the Appendix.

If ITR falls within its control limits without a consistent pattern, no problems are indicated and further action is unnecessary until the next stockout or review period. If ITR is outside the control limits, the manager should check to see if demand is in control. If not, it is likely that the performance deviation observed in the ITR measure is actually due to a change in demand. According to the flowchart in Figure 1, this situation leads to the conclusion that a fitness-related problem exists and corrective action must be taken. The manager may want to adjust the order quantity, safety stock level, and/or reorder point to reflect the changed operating environment.

If the demand is in control, then the cause of the out of control condition for the ITR measure must be lead time related or operations-related. For example, the inventory level could fall significantly below the predetermined level owing to a failure to place a replenishment order. This low inventory level could cause the ITR to exceed its upper control limit even though demand falls within its control limits.

This procedure would normally be computerized and implemented using data that is readily available in most inventory management systems. Its primary benefit is that it would automate the process of monitoring the inventory system and thereby require little extra time or effort from managers. When problems do occur the system would generate an exception report and alert management to the nature of the problems and their possible causes.

A Sample Problem

Consider an item whose inventory is controlled by using a reorder point system (remember every inventory item will be monitored in the same way). The average demand rate for the item is 20 units per day (5,000 units/year) and the standard deviation of daily demand is two units. With an average lead time of four days, the standard deviation of demand over replenishment lead time is four units. Using a safety factor of three standard deviations the safety stock would be 22 units (21.63) because of the demand and lead time variation (standard deviation of lead time is 0.3 days) as shown in[6]. Using the cost factors shown in Table I the EOQ for the item is 300 units. Additional information about this item is summarized in Table I.

Establishing Control Limits

The control limits for demand are determined using the average and standard deviation of demand during the order cycle. The standard deviation over the order cycle is found by multiplying the standard deviation of daily demand by the square root of the length of the order cycle. Thus, standard deviation is 7.746 (2×15). According to (1) and (2) in the Appendix, these parameters yield upper and lower control limits for demand of 323 and 277 units, respectively. It should be noted that the demand during the order cycle can go outside the control limits without showing any unusual changes in the daily demand rate. The control chart for demand is shown in Figure 2.

(1) Order cycle	(2) Day number	(3) Lead time	(4) Cycle days	(5) Average inventory	(6) Actual demand	(7) Actual turns	(8) Units short	(9) Cycle demand
1	15	4	15	164.13	307	31.17		307.00
2	29	4	14	164.07	280	30.47		300.00
3	45	4	16	174.44	306	27.41		286.88
4	61	5	16	154.50	334	33.78	2	313.13
5	75	4	14	153.64	267	31.03		286.07
6	90	4	15	166.87	308	30.76		308.00
7	104	4	14	171.50	280	29.15		300.00
8	120	4	16	177.63	317	27.89		297.19
9	134	4	14	169.43	298	31.41		319.29
10	148	4	14	168.36	308	32.67		330.00
11	162	4	14	156.57	305	34.79		326.70
12	175	4	13	163.69	283	33.25		326.54
13	189	4	14	171.93	307	31.89		328.93
14	204	5	15	150.43	326	36.05	1	326.00
15	217	4	13	147.00	283	37.02		326.54
16	232	5	15	144.27	326	37.66	10	326.00
17	245	5	13	140.15	283	38.83		326.54

Ordering cost	= $90 per order	Average lead time	= 4 days
Carrying cost	= $10/unit/year	Standard deviation (lead time)	= 0.3 days
Annual demand	= 5000 units	Safety stock	= 22 units
Average, daily demand	= 20 units	Reorder point	= 102 units
Standard deviation (demand)	= 2 units/day	EOQ	= 300 units

Table I. Summary Results from the Inventory Simulation

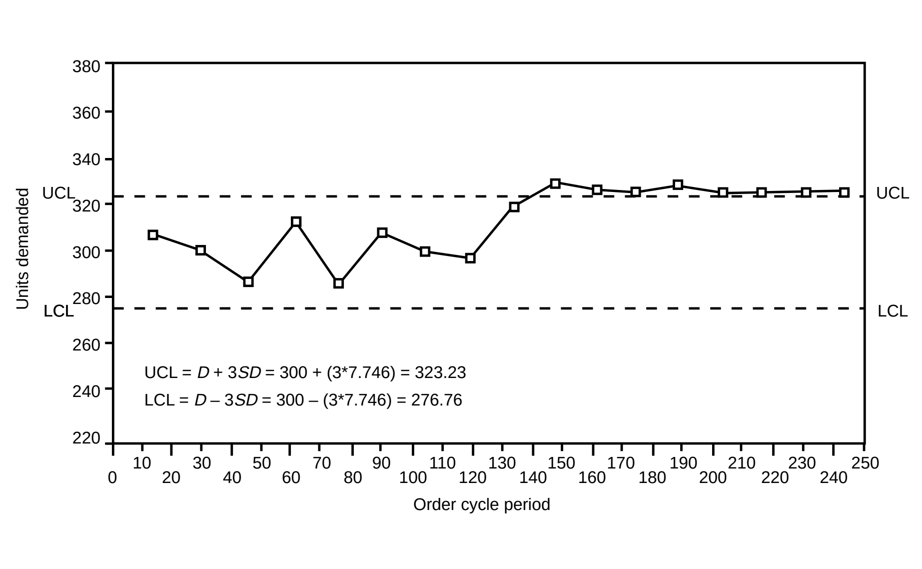

Figure 2.
Demand during the Order Cycle

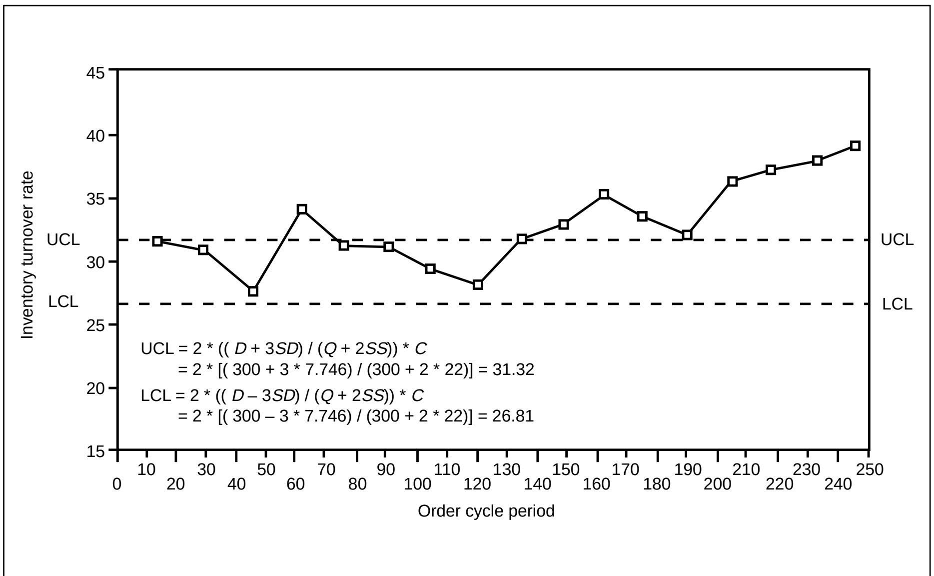

Figure 3.
Inventory Turnover for Each Order Cycle

The expected ITR is calculated by multiplying the turnover rate per order cycle by the expected number of cycles per year. The expected number of cycles per year is 16.67[(250 days per year)/(15 days per order cycle)]. Thus, according to (3) in the Appendix, the expected ITR is 29.08 times per year[2 * (300/(300 + 2 * (22))) * 16.67]. Using (4) and (5) in the Appendix, the values of the upper and lower control limits for ITR are 31.33 and 26.82, respectively. The specific calculations used to arrive at these values and the control charts are shown in Figure 3.

Simulating the Inventory Control and Monitoring System

Using the parameters at the top of Table I, a 250-day simulation of the inventory system was performed. The demand was simulated on a daily basis and an order of 300 has just arrived on day 1 and is used with the starting inventory of 25 to cover demand until the next order arrives. In the example shown in Table I, the first order cycle was exactly 15 days as planned. Thus, the cycle demand of 307 units was the same as the actual demand. The inventory turnover rate was 31.17 ((307/164) $\times$ (250/15)). The second order cycle ended in only 14 days; actual demand was 280 units during the cycle. Average inventory was 164.07 units. Cycle turnover ratio would be 1.706 (280/164.07); actual (annualized) turnover was 30.47, which is the product of the number of cycles per year 17.86 (250/14) and the cycle turnover rate 1.706. The normalized cycle demand is 300 units (280/14 $\times$ 15 days). In both cycles, the inventory turnover rates and the cycle demands are within their respective control limits, and there were no stockouts in the system.

This normalized approach to determining turnover rate and cycle demand provides accurate and comparable performance measures, since each turnover calculation is adjusted for the number of periods in the cycle. Because the approach adjusts to normal changes in demand and different starting inventory positions, it should generate few false signals. The method also provides the indirect benefit of an ABC classification for monitoring inventory, since A items, which tend to have shorter reorder cycles, are examined more often.

In the simulated reorder point system, demand and lead time were gradually increased over time to create sample out-of-control situations. As shown in Table I, the inventory system does not experience a stockout or generate an out-of-control indication for demand or turnover until the cycle ending on day number 61. In that cycle, a stockout of two units occurred, thus, the cycle demand rate should be checked. Since the cycle demand rate, 313.13, is within the control limits, the next step is to check whether the replenishment order was placed on time. The record shows that the order was placed on time, but that it did not arrive on time. Therefore, it can be shown that the stockout was caused by lead time and/or supplier-related problems. Also note that, in this instance, the inventory turnover rate of 33.78 is out of control owing to the low average inventory level caused by the late order.

The next out-of-control occurs in the order cycle ending in period 134 when ITR is out of control. On further investigation it indicates that demand is also nearly out of control. In this case the monitoring method is giving early warning about a problem with demand. During the next cycle ending on day 148, the inventory turnover rate (32.67) exceeds its upper control limit (31.33) again. The subsequent check of the cycle demand rate shows that it now also exceeds its upper control limit (323.23). These are indications that the demand rate has shifted upward and thereby caused the inventory turnover rate to go out of control. In fact, Figures 2 and 3 show that demand and ITR are out of control for the rest of the year and even create a shortage in two order cycles. However, the monitoring system would require that management further investigate and determine the causes of the out of control condition. Management would then take any necessary action to correct the problem.

Summary and Suggestions for Future Research

This article developed a conceptual model for detecting and diagnosing problems in reorder point systems. It is based on the notion of control charts and, thus, provides prompt and continual feedback regarding system performance. By monitoring stockouts and control charts for demand and inventory turnover, the inventory manager can isolate the causes of system malfunctions. In this fashion, problems can be identified and resolved quickly and the inventory system will generally be functioning as intended.

This research has two particularly intriguing extensions. The first is to expand the model proposed here to include other inventory control systems, such as periodic review and dependent demand systems. The other is to add correcting mechanisms to the model that would enable it to recommend and, possibly, implement solutions to diagnosed problems. For example, in the independent demand system discussed herein, order quantities and reorder points could be adjusted dynamically in response to changes in demand levels and lead time lengths.

References

1. Aggarwal, S.C., "A Review of Current Inventory Theory and Its Applications," *International Journal of Production Research,* Vol. 12, 1974, pp. 443-82.
2. Silver, E.A., "Operations Research in Inventory Management: A Review and Critique", *Operations Research*, Vol. 29 No. 4, July-August 1981, pp. 628-45.
3. Brown, R.G., *Decision Rules for Inventory Management,* Holt, Rinehart and Winston, New York, NY, 1967.
4. Tersine, R., *Principles of Inventory and Materials Management,* North-Holland, New York, NY, 1988.
5. Eilon, S. and Elmaleh, J., "Adaptive Limits in Inventory Control," *Management Science,* Vol. 16 No. 8, April 1970, pp. 533-48.
6. Pope, J. and Ardalan, A., "Evaluating Order Point Inventory Systems," *Journal of Purchasing and Materials Management,* Vol. 24 No. 2, 1988, pp. 27-32.

Appendix: Techniques of Establishing Control Limits

Control Limits for Demand

In an reorder point system, the length of the order cycle is adjusted automatically as the demand rate changes while the average demand during the order cycle remains relatively constant. This means that any increase or decrease in the demand rate will be reflected in the length of the order cycle, not in the demand during the order cycle. Thus, in order to develop a meaningful control chart for demand, it must be expressed in terms of the demand that would be expected during a "normal length" order cycle. This can be accomplished by establishing control limits as follows:

$$\text{Upper Control Limit Demand } (D) = D + 3\sigma \qquad (1)$$

Lower Control Limit Demand (D) = $D - 3\sigma$ (2)

where:

D = average demand during the order cycle

σ = standard deviation of demand during the order cycle.

Control Limits for Inventory Turnover Rate

An expected inventory turnover rate (ITR) is calculated as the ratio of average annual demand to average annual inventory level. Using demand, safety stock, and order quantity the expected ITR rate can be expressed as:

$$\text{Expected ITR} = (D / (1/2(Q) + SS)) \times C$$

$$= 2(D / (Q + 2SS)) \times C \quad (3)$$

where:

D = average demand during the order cycle in units

Q = economic order quantity in units

SS = safety stock in units

d = average daily demand in units

TBO = time between orders in days (Q/d)

L = year length (250 days)

C = the optimal number of order cycles per year (L/TBO).

After determining the value of the expected ITR, the upper and lower control limits of the turnover rate must be computed. These limits define the normal range of fluctuation in the ITR and are set by considering the fluctuation range of demand, an environmental variable. These are defined as:

$$\text{Upper Control Limit (ITR)} = 2((D + 3\sigma) / (Q + 2SS)) \times C \quad (4)$$

$$\text{Lower Control Limit (ITR)} = 2((D - 3\sigma) / (Q + 2SS)) \times C. \quad (5)$$

This article has been cited by:

1. Nan Chen, Shiyu Zhou. 2015. CUSUM Statistical Monitoring of M/M/1 Queues and Extensions. *Technometrics* **57**:2, 245-256. [CrossRef]
2. Francesco Costantino, Giulio Di Gravio, Ahmed Shaban, Massimo Tronci. 2015. A real-time SPC inventory replenishment system to improve supply chain performances. *Expert Systems with Applications* **42**:3, 1665-1683. [CrossRef]
3. Ranjan Bhattacharya, Susmita Bandyopadhyay. 2011. A review of the causes of bullwhip effect in a supply chain. *The International Journal of Advanced Manufacturing Technology* **54**:9-12, 1245-1261. [CrossRef]
4. C.Y. Lam, W.H. Ip. 2011. A customer satisfaction inventory model for supply chain integration. *Expert Systems with Applications* **38**:1, 875-883. [CrossRef]
5. Jui-Chang Cheng, Chao-Yu Chou. 2008. A real-time inventory decision system using Western Electric run rules and ARMA control chart. *Expert Systems with Applications* **35**:3, 755-761. [CrossRef]
6. H.T. Lee, J.C. Wu. 2006. A study on inventory replenishment policies in a two-echelon supply chain system. *Computers & Industrial Engineering* **51**:2, 257-263. [CrossRef]
7. Gyula Vastag, D. Clay Whybark. 2005. Inventory management: Is there a knock-on effect?. *International Journal of Production Economics* **93-94**, 129-138. [CrossRef]
8. Tim HaslettMonash University, Melbourne, Australia, and Charles OsborneMonash University, Melbourne, Australia. 2000. Local rules: their application in a kanban system. *International Journal of Operations & Production Management* **20**:9, 1078-1092. [Abstract] [Full Text] [PDF]