

27th CIRP Design Conference

Multi-layer graph theory utilisation for improving traceability and knowledge management in early design stages

David Ríos-Zapata^{a,b,*}, Jérôme Pailhès^b, Ricardo Mejía-Gutiérrez^a

^aDesign Engineering Research Group (GRID), Universidad EAFIT, Carrera 49 # 7 Sur - 50, Medellín, Colombia

^bArts et Métiers ParisTech, I2M-IMC, UMR 5295, F-33400 Talence, France

* Corresponding author. Tel.: +57-4-261-9500; Ext. 9712. E-mail address: rmejia@eafit.edu.co

Abstract

Decision making processes in design often challenges designers to prioritise specifications and variables in order to develop solutions that are closer to the product's requirement goals. Concerning to support their decisions, different tools and methods are used by engineers and designers allowing to reduce uncertainty in design. Nevertheless, many of these decision support systems are focused in late design stages, such as detailed design and manufacturing design, even if the possibility to influence a new product is higher in early stages. The issues regarding to those situations are often associated to design processes related to multi-physics design, where the modification of geometric-related variables might affect the performance of the solution, and the analysis of tracking the influence of the modifications might generate reprocessing and loses of time, specially when those relations are tricky and are not easily identifiable by analysing equations and a manual analysis of requirements must be performed. This article is centred in proposing a traceability model for early design stages based in graph theory. The proposal supports the information generated in design, from the input requirements (linguistic field) up to mathematical modelling and variables definition (real numbers field). This information is arranged into different layers, allowing a multilevel approach in terms of information management. The model also features a novel solution for weighting vertex in graph model, featuring a model that balances the direction of improvement, the importance and flexibility of any specification and how its behaviour will affect the design variables associated to it. The goal of the proposed model is to offer to designers, since the conceptual design stage, a method that can show automatically the level of correlation between any pair variables and specifications by the use of information trees and featuring chains that can connect them whether there is or not a connection via equations.

© 2017 Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license

(<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

Peer-review under responsibility of the scientific committee of the 27th CIRP Design Conference

Keywords: Traceability in design, traceability in early design stages, information management model, graph theory in design, decision support system

1. Introduction

One of the more natural aspects related to decision making in design recalls in coming ahead any unexpected interaction, which means, changing one *design variable* will not affect in a negative matter any other *requirement*. Managing this type uncertainty in early design stages is one of main facets to study within the XXI century demands.

In this connection, one of the approaches to handle with uncertainty management is increasing the traceability of the information at early design [1]. Those approaches stimulate the development of new technologies for early design stages, where the appearance of new tools it is being a constant over the last few decades. Likewise, its usage is highly motivated by the automation of different task at those design stages [2], and offering saves in time and money as well [3].

This article is centred in proposing a traceability model to be used in early design stages, offering connections within the evolution of the design parameters from marketing inputs, where

inputs are in a linguistic manner (e.g. "*the product must be big*"), up to variables definitions (length, diameter, etc.). The purpose of this model is to generate sensibility and correlations index between the design variables and the success criteria of each requirement.

2. State of the art

For over a decade the development of tools for supporting early design stages, specially since the lack of tools at those stages is evident [4]; also, the development of tools and methods had empower to increase the success rate in market of new products up to 60% [3].

Associated to tools, also different design methodologies had also improved the work, allowing time reductions and better team work [5]. Under the frame of this article, three thematic areas are related: design methodologies, traceability and uncertainty.

2.1. Design methodologies

Product design can be divided in four principal stages: clarification of the task, conceptual design, embodiment design and detail design [6]. Under the frame of this article, there will be considered as early design the first three stages, up to the definition of design equations, but not the final value of the geometric entities that are represented as variables in the equations. Also, in those stages it is important to recall how the information evolves from linguistic inputs, to fuzzy numbers and finally into real numbers [7].

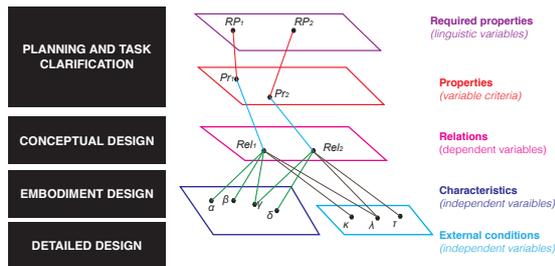


Fig. 1. Information evolution through design processes

From Figure 1, the proposal is centred into using the generated information from well know design methodologies in order to create a traceability tree to empower decision making in design. Within this frame of this research, the following nomenclature will be used: linguistic domain coming from marketing requirements will be called *Required properties*; *Properties* is the product behaviour that response to what designers want; the equations of the product will be known as *Relations*; finally the design variables are divided in two, *Characteristics* which are the parameters that can be directly influenced by the designers and *External Conditions* that can not be influenced and are defined by the external environment [8].

2.1.1. Clarification of the task

Related to the work made by the designer, several task are accomplished in order to traduce that linguistic information into technical requirements. For instance, tools like Quality Function Development (QFD) are used to perform this commitment [9]. Also, in terms of generating specifications of the product, functional analysis can be used in order to exploit the relationship of the product with the environment for this purpose [10].

In this phase, is very important for the model that a Functional Analysis be performed, and all the specifications written based in functions (i.e. functions result from *octopus diagrams* [10]). This will generate the $CdCF_0^1$. After designers generate the specifications of the product, the QFD must be performed. This will allow to relate the requirements that are the result of marketing and user understanding with the technical specifications that the product must assure. This CdCF is related to the design criteria to each specification of the product.

¹French for Cahier des Charges Fonctionnel, a list with the specifications of the product.

2.1.2. Conceptual design

The conceptual design is centred in Pahl & Beitz approach [6]. Nevertheless is important to consider the important to evaluate each function using the CTOC approach [11]. This approach treats each energy flow as: Converter-Transmitter-Operator-Control. Its usage is a key in order to simplify the functions by understanding how energy is transformed and which are the surfaces that act in the process. The goal of these stages is generating a FBD (Function Block Diagram) containing all the fluxes of energy, matter and information.

2.1.3. Embodiment design

In the edge of both phases, designers answer the relations that will engage the behaviour of the solution. Next, the CPM/PDD can be performed [12], generating connections between the equations and the variables and populating with equations each block of the FBD.

2.2. Traceability in early design

Within the last decades, different models had been proposed for early design. Baxter et al. had defined a traceability framework focused in optimising design solutions by analysing the performance of certain requirements [13]. Nevertheless at linguistic levels (requirements definition) many of those information management models deal with poor data traceability [14], and usually the information is only stored at a specific location but it is not exploited [15].

This leads to define the importance of developing tools that can assures high level of detail in the creation of the information links at early design stages [1]. Finally, according to Ouertani et Al., a good traceability tool should identify the dependence of the design of terms of variability, sensitivity and integrity [16].

2.3. Uncertainty in early design

Uncertainty is hooked up to decision making in design as one of the main characteristic of the profession itself; designers must somehow anticipate how their decisions will affect the performance of the product [17]. Naturally, design methodologies are developed to reduce this lack of awareness in decision making [7].

In terms of defining the type of information generated and shared, and understanding who, which, why and when would that information needed by other members of development team, there is a further complexity of design management. And whenever that information is not available, the level of uncertainty is increased because of the assumptions that are needed to be made [18].

For design activities, two types of uncertainties can be described: aleatory and epistemic. The first type is related to the natural randomness of the product characteristics and physical properties. Epistemic is related to the imprecision that happens because lack of knowledge [19]. Moreover, epistemic can divided into five categories: model, phenomenological, behavioural, ambiguity and interaction [20].

In order to treat uncertainty, Malmiry et Al. had defined a functional modelling approach, for early design that is handles both types of uncertainty by the use of CPM/PDD modelling [12]. Within the interpretation of functions, and its definitions into equations, this approach manages uncertainty by

analysing: characteristics, properties, relation, external conditions, modelling conditions and required properties. Finally, two types approaches can be conducted in order to manage different types of uncertainty: analysis and synthesis[8,21] (See Figure 2). *Analysis*: based on known characteristics, the properties can be determined. Several approaches are used to predict the product performance in an experimentally manner, such as CPM use [12] and optimisation of characteristics [22]. *Synthesis*: based on required properties, the characteristics can be determined; in this connection the challenge of the designer is to determine appropriate solution patterns that meets customer needs.

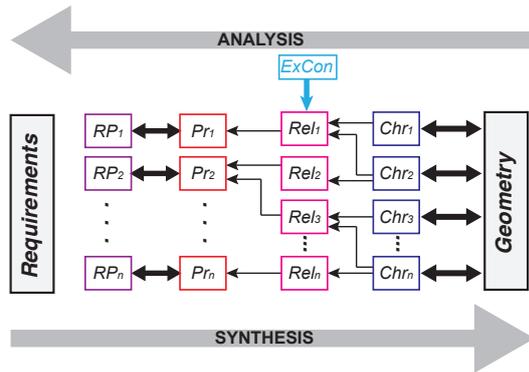


Fig. 2. Design process strategies. Adapted [8]

3. Graph construction

The proposal is not intended to be a framework for product design, opposite, it takes information that comes as the result of a design process. This information enters to the system in three matrices. From those matrices the whole graph is build.

3.1. Required Properties to Properties

Table 1 relates the first input to the system. This matrix will make the connection between the first two levels: Required Properties to Properties. Table 1 relates only an extract from a full QFD matrix, and shows only what is relevant for the proposal to work. The extracted matrix features the correlation part, which indeed makes a connection between both parts *Required Properties* in linguistic field and *Properties*.

Table 1. Matrix 1: QFD extract

	Imp.	Pr ₁	Pr ₂	Pr ₃
Rq 1	5	●		▲
Rq 2	3		●	
Rq 3	2			●
		50.4%	26.8 %	22.8 %

3.2. Properties to Relations

In order to use the model, it is very important to build the Properties list based from a Functional Analysis approach (oc-topus diagrams). This will allow to have Properties that match

with functions. Table 2 relates an example of a Properties list made using a *CdCF* notation. To this table, it is added a new column, *Function Map*, which is important to remark the tracking of each specification.

Table 2. Product Design Specifications list

N	Spec.	k	Metric	Level	F	Fn Map
1	Sp 1	5	m	(2,3) ± 0.01	F1	Fs1
2	Sp 2	3	°C	(25,28) ± 1	F2	Fc1, Fs1
3	Sp 3	4	m	(0.1,0.2) ± 0.001	F0	Fc2

Then, by the use of a CPM/PDD approach [12], based on each function, several equations that support the product design could be conceived. For the model to work, it is necessary that the user construct a matrix that relates the relationship between the functions and the generated equations.

Finally, a matrix multiplication between *CdCF* and the Functions-Equations matrix will allow to connect **Properties** with the **Relations** that allow the product to fulfil the design conditions. Since the functions vector match in both arrays, the result of the multiplication will construct a relationship between both needed items. This can be watched in Table 3.

Table 3. Properties to Relations

$$\begin{bmatrix} PR_1 \\ PR_2 \\ PR_3 \end{bmatrix} \begin{bmatrix} FS1 & FC1 & FC2 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} Rel_1 & Rel_2 & Rel_3 & Rel_4 \\ FS1 & 1 & 0 & 0 \\ FC1 & 0 & 1 & 0 \\ FC2 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} PR_1 \\ PR_2 \\ PR_3 \end{bmatrix} \begin{bmatrix} Rel_1 & Rel_2 & Rel_3 & Rel_4 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

3.3. Relations to Characteristics

In this section is where the use of CPM/PDD becomes very important in order to develop a mathematical model that does not take a lot of time to compute. The use of CPM/PDD, connected with CTOC approach, allow to divide each equation in several sub-equations.

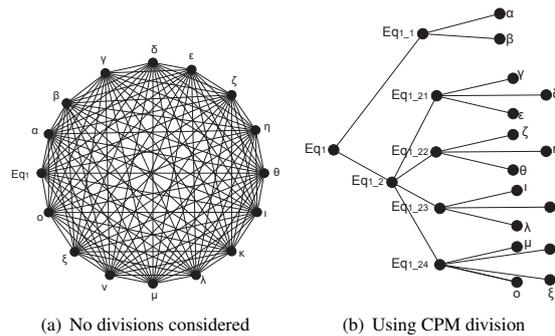


Fig. 3. Equation representation

Considering Equation 1², this equation might be represented by two different approaches. The first approach is to not divide

²Equations can be composed by characteristics and external conditions

the equations in different sub-equations. When the equation has many elements (i.e. 16 elements), this leads to high computation time. In Figure 3a there is a plotted equation-variable system without considering divisions.

$$Eq_1 = \frac{\alpha - \beta}{\frac{\gamma}{\delta * \epsilon} + \frac{\zeta}{\eta * \theta} + \frac{\iota}{\kappa * \lambda} + \frac{\mu * \nu}{\xi * \omicron}} \quad (1)$$

In terms of computation time, any calculation between the equation will develop $n!$ possibilities, i.e. $15!$. Nevertheless, in CPM/PDD the equation is divided into sub-equations, generating systems that consume less computation time and allow a better tracking in the system; reducing uncertainty as well.

4. Synthesis Model Proposal

In order to verify the influence between characteristics and properties, an analysis-synthesis model is being proposed. This model, empowered by the use of graph allows to verify if there is a connection between any characteristics and properties; also, allows to guide designers to understand how modifying characteristics disturb or aids different properties.

The first part of the model, in the analysis section, designers are invited to define for each characteristic its behaviour in terms of the tendency that each variable must have in order to accomplish the desired performance. That is to say the designer must define in this stage, in terms of design intervals, if the variable must be centred in the range or it will be close to the upper or lower-bound of the range. This can be watched in Figure 4.

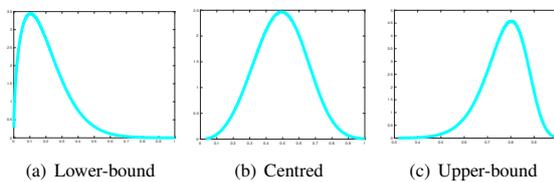


Fig. 4. Variables distribution

The distributions defined to each behaviour were developed using Weibull distributions; for Figure 4a, lower-bound was defined using a Scale of 1 and a Shape of 1.5. This will assure that most of the possible numbers will have a tendency close to the minimum value. For Figure 4b, centred was a Weibull with a Scale of 1 and a Shape of 3.5, which allows a centred distribution with short tails. For Figure 4c, upper-bound had a Scale of 1 and a Shape of 10, maximising the tendency to the values close to the maximum value.

Defined this distribution, a Monte Carlo simulation is performed in order to perform a sensitivity analysis, both, local (between relations, sub-relations with characteristics) and global (between properties and characteristics). This process allows to construct a full traceability tree with its vertex weight information.

Afterwards, the synthesis section can be prepared. It is important to recall that the traceability tree holds two types of chains. The first type of chain represents the direct connections

between properties and characteristics: following paths made by the vertexes created during the tree population following the CPM/PDD methodology; a in Figure 6a, the bold lines represents single chains (direct relations). The second type might be considered as complex chains, and those represent indirect connections between properties and characteristics (see dashed line in Figure 6b).

In the model, single chains are used for calculating property's gradients. This is doing by obtaining the first-degree derivative of the property equation respect its whole characteristics, featuring analysis intervals for any characteristic range. This is explained in detail in section 5.

Finally, one of the features of the synthesis model is the generation of these second type of chains, which is made by the implementation of minimum spanning tree algorithm, based in Dijkstra's shortest path algorithm [23]. This allows to generate equations that relate characteristics even if they do not make part of the property relation.

5. Case study

A brief case study is introduced. It is intended to design a portable cooler. Its marketing inputs derive into 3 Required Properties and 5 Properties (See Table 4). After conducting its design process, 9 characteristics and 4 external conditions were determined. The full traceability tree of the model can be watched in Figure 5.

Table 4. Properties list and function map

Pr number	Property	FS1	FC1	FC2	FC3
Pr_1	Internal temperature	1			
Pr_2	Time to heat	1			
Pr_3	Weight		1		
Pr_4	Internal volume				1
Pr_5	External length			1	

Within its design process, three properties are going to be considered in order show how the proposed method can be used to find paths and calculate local and global sensibility analysis: $Pr_1 = Internal\ temperature\ (T_{int})$, $Pr_4 = Internal\ volume\ (Vol_C)$ and $Pr_5 = External\ length\ (L_{ext})$.

$$Rel_2 - > Q_{conv} = \frac{T_{ext} - T_{int}}{\frac{L_A}{K_A * A_A} + \frac{L_B}{K_B * t_B} + \frac{L_C}{K_C * A_C}} \quad (2)$$

Following the CPM/PDD design process Pr_1 is related to the relation listed in Equation 2, that represents the physical phenomena of heat transfer in a cooler³. Also Pr_4 related to Equation 3, representing C the internal dimensions of the cooler and Pr_5 is represented to Equation 4 and its success criteria is given by ergonomics requirements.

$$Rel_3 - > Vol_C = L_C * W_C * h_C \quad (3)$$

³The isolation system selected is described as *sandwich*, composed by an external wall, thermal insulation, and internal wall

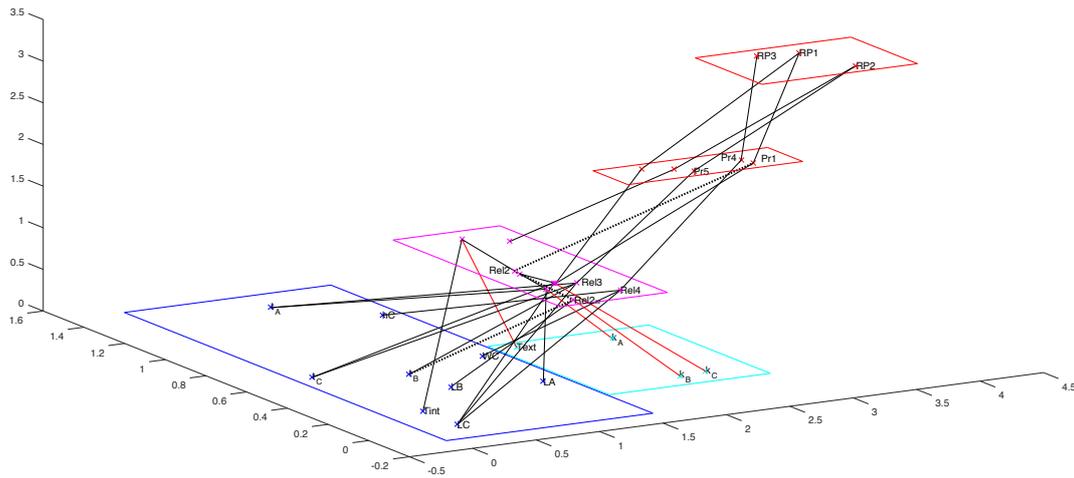


Fig. 5. Traceability tree of a portable cooler

A decision making challenge will be considered. For example it is requested to increase the heat isolation of the cooler, with the condition of maintaining materials; that will determine that heat transfer coefficients remain constant (treating them as external conditions). By doing so, a single criteria global sensibility analysis in for Pr_1 is performed.

$$Rel_4 - > L_{ext} = L_C + 2t_C + 2t_B + 2t_A \quad (4)$$

This section, the analysis section, grants as a result that the best alternative to increase isolation is increasing the thermal isolator thickness (t_B). By analysing equations, it can be seen that this characteristic may also affect Pr_5 , but no further conclusion about the interaction of this characteristic with other properties, such as volume, can be taken.

Those type of situations increase uncertainty in decision making at early design: probably experienced engineers know that the increasing t_B will improve the cooling performance of the cooler, but will affect the volume or external length; but in complex design, with hundreds of variables and characteristics, these analysis should not be based on experience. In this case, the proposal can track if there is any connection between any pair of characteristics. This can be watched in Figure 6.

Regarding to the interaction between t_B with the selected properties, the analysis section can be performed. Because the analysis part reported t_B as the characteristic with more possibility to influence, this characteristic will be analysed.

The first part is to calculate the derivative of Pr_1 respect t_B , this can be watched in Equation 5. Since t_B also has a direct connection with Pr_5 . The derivative should be calculated as well.

$$\frac{\partial Pr_1}{\partial t_B} = \frac{K_A^2 K_A^2 K_B K_C^2 L_B t_A^2 t_C^2 (T_{ext} - T_{in})}{(t_B K_B (t_A K_A L_C + t_C K_C L_A) + K_A K_C L_B t_A t_C)^2} \quad (5)$$

After evaluating Equation 5, between 0.01 – 0.03, it is found that the values of ∂Pr_1 vary between 0.914⁴ to 0.45 ; for the same range, $\partial Pr_1 \partial t_A$ vary between 10.84 to 1.67.

$$\frac{\partial Pr_1}{\partial t_B} |_{t_B=[0.01,0.03]} = \frac{0.002107}{(t_B + 0.038011)^2} = [0.45, 0.914]$$

Regarding to Pr_5 , $\partial Pr_5 \partial t_b$ its result is 2, presenting the same value with t_A and t_C . For the enquiry of both properties. It is important to recall the usage of the intervals of solution of the characteristics, but the criteria range as well. In this case, t_B derivative has lower and steady values for both parameters: this represents that varying within its domain has a favourable response in the properties. In the case of t_A , the response for Pr_5 presented any difference with t_B 's behaviour, but in terms of evaluating Pr_1 , the difference of its response is gigantic endangering the property to keep in target criteria.

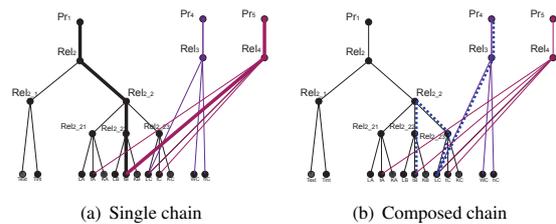


Fig. 6. Types of chains within traceability tree

Finally, the designers are in charge to manually vary characteristics and manually looking how any change affect the properties. The added value of the model is guiding designers to detect which characteristics offers a better improvement to a property, by the global sensibility analysis. Also, the model is

⁴0.914 when t_B is 0.01 meters

able to calculate multi-influenced criteria, whenever there is a characteristic that affects several properties. The procedure for single chains (Figure 6a) can be made as an analytical process and was just described.

Nevertheless, for complex chains (Figure 6b), there is not any equation that can directly connect the characteristic with the property, in this situation Pr_4 to t_B . The procedure followed was to include the evaluation of $\frac{\partial Pr_4}{\partial t_B}$ and with its values, for re-writing Equation 2 in terms of L_C , the shared characteristic between both properties (See 6). Finally the influence of t_B and Pr_4 is found as -617.91 to -68.65 (For making a comparison, t_A is between -179.2 to -19.91 ; same 10% range).

$$\frac{\partial Pr_4}{\partial t_B} = \frac{-H_C K_C L_{BtC} W_C}{t_B^2 K_B} \quad (6)$$

6. Conclusions and further research

One of the main conclusions is that the proposal is able to build a traceability tree only with information that comes from well-known design process. Yet, it still is limited to the experience of the designers team in order to conduct a CPM/PDD approach. Designers with difficulties in this modelling might create information that will construct distorted trees. Still, the model can provide useful information, in an automatically manner, for determining degrees of correlations between any pair of characteristics-properties. In terms of decision-making, this proposal can be helpful in order to reduce epistemic uncertainty by providing a better understanding of model interactions.

The analysis of the derivatives aids to understand the behaviour of the characteristics in terms of variation. For instance, t_A 's derivative presents an unsteady variation for Pr_1 generating uncertainty whenever designers want to do a change; a tiny change will affect the property a lot. On the other hand, when the derivative difference is not high (percentages evaluation), that indicates that performing changes will not impact the property dramatically.

The further research should be centred in three aspects: connecting the model to cloud systems, in order to empower traceability along the whole life cycle, improving communication with detailed design and exploring how these traceability systems can influence decisions. Also, improving information storage in order to achieve deeper levels in knowledge engineering.

Generating of a risk classification based in the whole traceability tree: automatically informing which characteristics are more critic to properties performance.

Finally, the major further research is centred in how, based in the complex chain created Dijkstra algorithm, a sequence including the vertexes weight's can be created. So far, several broadcasting algorithms featuring domination trees and graphs factorisation can be found in literature [24,25] but still is necessary study weight integration. The goal is to offer a model that calculates correlations no matter type of chain.

References

[1] Königs, S.F., Beier, G., Figge, A., Stark, R. Traceability in systems engineering—review of industrial practices, state-of-the-art technologies and new research solutions. *Advanced Engineering Informatics* 2012;26(4):924–940.

[2] Robertson, B., Radcliffe, D. Impact of CAD tools on creative problem solving in engineering design. *Computer-Aided Design* 2009;41(3):136–146.

[3] Valle, S., Vázquez-Bustelo, D. Concurrent engineering performance: Incremental versus radical innovation. *International Journal of Production Economics* 2009;119(1):136–148.

[4] Wang, L., Shen, W., Xie, H., Neelamkavil, J., Pardasani, A. Collaborative conceptual design—state of the art and future trends. *Computer-Aided Design* 2002;34(43):981–996.

[5] Prasad, B. Concurrent engineering fundamentals- Integrated product and process organization. Upper Saddle River, NJ: Prentice Hall PT; 1996.

[6] Pahl, G., Beitz, W., Feldhusen, J., Gote, H. Engineering design: A systematic approach. Springer Verlag; 2007.

[7] Giachetti, R.E., Young, R.E., Roggatz, A., Eversheim, W., Perrone, G. A methodology for the reduction of imprecision in the engineering process. *European Journal of Operational Research* 1997;100(2):277–292.

[8] Weber, C. Cpm/pdd—an extended theoretical approach to modelling products and product development processes. In: *Proceedings of the 2nd German-Israeli Symposium on Advances in Methods and Systems for Development of Products and Processes*. 2005, p. 159–179.

[9] Prasad, B. Review of qfd and related deployment techniques. *Journal of manufacturing Systems* 1998;17(3):221.

[10] Scaravetti, D., Nadeau, J.P., Pailhès, J., Sebastian, P. Structuring of embodiment design problem based on the product lifecycle. *International Journal of Product Development* 2005;2(1):47–70.

[11] Pailhès, J., Sallaou, M., Nadeau, J.P., Fadel, G.M. Energy based functional decomposition in preliminary design. *Journal of mechanical design* 2011;133(5):051011.

[12] Malmiry, R.B., Dantan, J.Y., Pailhs, J., Antoine, J.F. A product functional modelling approach based on the energy flow by using characteristics-properties modelling. *Journal of Engineering Design* 2016;27(12):817–843.

[13] Baxter, D., Gao, J., Case, K., Harding, J., Young, B., Cochrane, S., et al. A framework to integrate design knowledge reuse and requirements management in engineering design. *Robotics and Computer-Integrated Manufacturing* 2008;24(4):585–593.

[14] Igba, J., Alemzadeh, K., Gibbons, P.M., Henningsen, K. A framework for optimising product performance through feedback and reuse of in-service experience. *Robotics and Computer-Integrated Manufacturing* 2015;36:2–12.

[15] Zheng, C., Bricogne, M., Le Duigou, J., Eynard, B. Survey on mechatronic engineering: A focus on design methods and product models. *Advanced Engineering Informatics* 2014;28(3):241–257.

[16] Ouertani, M.Z., Baïna, S., Gzara, L., Morel, G. Traceability and management of dispersed product knowledge during design and manufacturing. *Computer-Aided Design* 2011;43(5):546–562.

[17] Clarkson, P.J., Simons, C., Eckert, C. Predicting change propagation in complex design. *Journal of Mechanical Design* 2004;126(5):788–797.

[18] Danilovic, M., Sandkull, B. The use of dependence structure matrix and domain mapping matrix in managing uncertainty in multiple project situations. *International journal of project management* 2005;23(3):193–203.

[19] Malak, R.J., Aughenbaugh, J.M., Paredis, C.J. Multi-attribute utility analysis in set-based conceptual design. *Computer-Aided Design* 2009;41(3):214–227.

[20] Thunnissen, D.P. Propagating and mitigating uncertainty in the design of complex multidisciplinary systems. Ph.D. thesis; California Institute of Technology; 2005.

[21] Weber, C. Looking at dxf and product maturity from the perspective of a new approach to modelling product and product development processes. In: *The Future of Product Development*. Springer; 2007, p. 85–104.

[22] Collignan, A., Sebastian, P., Pailhes, J., Ledoux, Y. Optimization of product in dynamic design space and selection through the arc-elasticity concept. *International Journal on Interactive Design and Manufacturing* 2011;5(4):243–254.

[23] Dijkstra, E.W. A note on two problems in connexion with graphs. *Numerische mathematik* 1959;1(1):269–271.

[24] Cockayne, E.J., Herke, S., Mynhardt, C.M. Broadcasts and domination in trees. *Discrete Mathematics* 2011;311(13):1235–1246.

[25] Herke, S., Maenhaut, B. Perfect 1-factorizations of a family of cayley graphs. *Journal of Combinatorial Designs* 2015;23(9):369–399.